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A Compartmental Mathematical Model for the Control of Financial Debt Dynamics in Nigeria Using Leadership Intelligence Quotient (LIQ)

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ABSTRACT

The rapid expansion of public debt in developing economies requires mathematical frameworks capable of explaining the dynamic mechanisms underlying debt accumulation and identifying effective control strategies. This study develops a deterministic compartmental mathematical model to investigate the transmission dynamics of Nigeria's public debt by explicitly integrating Leadership Intelligence Quotient (LIQ) as institutional control variables. The model partitions the system into four interacting compartments representing the people, economic productivity, debt accumulation, and leadership quality, and using a system of nonlinear ordinary differential equations to describe their interactions. Fundamental qualitative properties of the model, including positivity, boundedness, and the existence of feasible equilibrium solutions, were established. The Next Generation Matrix approach was employed to derive the basic reproduction number, while local stability analysis, sensitivity analysis, and numerical simulations were conducted to evaluate the influence of model parameters on public debt dynamics. The analysis yielded a basic reproduction number of R_0 = 0.0667, indicating that the public debt system remains asymptotically stable under the prevailing control conditions. Sensitivity analysis identified debt transmission, governance inefficiency, and low leadership intelligence as the principal drivers of debt persistence, whereas improvements in governance effectiveness and leadership quality significantly reduced debt propagation and accelerated convergence toward equilibrium. Numerical simulations further confirmed that strengthening institutional quality suppresses debt accumulation and promotes long-term fiscal stability. The proposed compartmental framework provides a novel mathematical approach for analysing sovereign debt dynamics by explicitly integrating Leadership Intelligence Quotient (LIQ) into a deterministic nonlinear model. The findings demonstrate that improvements in governance quality and leadership capacity are critical to achieving long-term public debt sustainability in Nigeria. Accordingly, the study recommends sustained governance reforms, leadership capacity development, and institutional strengthening as essential strategies for maintaining long-term fiscal stability.

Introduction

1.1 Background to the Study

Public debt is an important instrument for financing development, infrastructure, and economic stabilization; however, excessive debt accumulation can create fiscal stress, weaken macroeconomic stability, and constrain economic growth. These concerns are particularly relevant in developing economies, where institutional weaknesses and governance challenges may increase fiscal vulnerability. Nigeria has experienced substantial growth in public debt over the past two decades, driven by persistent budget deficits, declining oil revenues, exchange-rate depreciation, inflationary pressures, and rising government expenditure. The International Monetary Fund (IMF) (2023) emphasizes the importance of sustainable debt managementin its fiscal monitor report and long term economic growth, while the World Bank (2023) highlights the role of institutional quality, governance effectiveness, and sound economic management in maintaining debt sustainability. The relationship between excessive debt and economic performance has received considerable attention. Reinhart and Rogoff (2010) associated high

debt burdens with weaker economic growth and increased fiscal vulnerability, while Krugman (1988) and Sachs (1989) explained that excessive indebtedness can discourage investment and generate long-term economic distortions through the debt-overhang effect. Traditional debt studies have largely relied on econometric approaches involving macroeconomic variables such as inflation, Gross Domestic Product (GDP), fiscal deficits, and exchange rates. Although these approaches provide valuable insights, they may not adequately represent the dynamic interactions among debt accumulation, economic productivity, governance quality, and leadership effectiveness (Samonas, 2015). Compartmental modelling offers an alternative framework for representing interconnected dynamic processes. Originally developed in epidemiology, compartmental approaches have also been applied to financial systems. Gai and Kapadia (2010), for example, demonstrated that financial disturbances can spread through interconnected institutions in a manner analogous to contagion. This perspective provides a basis for interpreting debt accumulation as a dynamic process influenced by economic, institutional, and leadership factors. Leadership quality and governance are also relevant to fiscal outcomes. Simonton (2006) and Winter (2005) emphasized the influence of leadership cognitive capacity on policy formulation, resource allocation, and strategic decision-making, while governance dimensions such as accountability, transparency, government effectiveness, and control of corruption are important to economic resilience and debt sustainability (Kaufmann, Kraay, & Mastruzzi, 2010). Despite these contributions, limited attention has been given to integrating debt dynamics, economic productivity, governance quality, and Leadership Intelligence Quotient (LIQ) within a unified compartmental framework. This study therefore develops a nonlinear compartmental mathematical model incorporating LIQ and the to analyse and control financial debt dynamics in Nigeria.

1.2 Statement of the Problem

Nigeria's increasing public debt burden remains a major concern for policymakers, economists, and development practitioners because persistent debt accumulation can threaten fiscal sustainability and economic stability. Despite various debt management initiatives and fiscal reforms, the country's debt burden continues to increase, creating a need for more effective approaches to understanding and controlling debt dynamics. Existing studies on public debt have predominantly focused on macroeconomic variables such as GDP growth, inflation, exchange rates, and government expenditure. Although these factors are important determinants of debt behaviour, such approaches provide limited attention to the roles of governance quality and leadership effectiveness in shaping debt accumulation and management. Furthermore, conventional debt models based mainly on econometric relationships may not adequately represent the dynamic interactions and transmission mechanisms among economic productivity, governance structures, leadership quality, and debt accumulation. Previous studies have examined debt sustainability (Reinhart & Rogoff, 2010), financial contagion (Gai & Kapadia, 2010), governance effectiveness (Kaufmann et al., 2010), and leadership performance (Simonton, 2006) largely as separate areas of investigation. However, limited attention has been given to integrating these dimensions within a unified mathematical framework capable of representing debt transmission, accumulation, and control. This gap limits understanding of how leadership intelligence and governance quality may interact with economic productivity and debt dynamics in Nigeria. Therefore, there is a need for a compartmental mathematical model that incorporates Leadership Intelligence Quotient (LIQ) and to analyse the transmission and control of financial debt dynamics. This study addresses this gap by developing such a model and examining its equilibrium states, debt reproduction number, stability properties, and the effects of leadership intelligence and governance quality on debt control.

1.3 Aim of the Study

The aim of this study is to develop a compartmental mathematical model for analysing and controlling financial debt dynamics in Nigeria through the integration of Leadership Intelligence Quotient (LIQ)

1.4 Objectives of the Study

The specific objectives are to:

i. Develop a compartmental mathematical model describing financial debt dynamics in Nigeria.
ii. Determine the equilibrium states of the model.
iii. Derive the debt reproduction number.
iv. Analyse the stability properties of the model.
v. Investigate the effects of leadership intelligence and governance quality on debt control.

1.5 Significance of the Study

This study contributes to the literature on debt sustainability by integrating Leadership Intelligence Quotient (LIQ) and into a compartmental debt-contagion framework. The model provides a quantitative approach for analysing debt dynamics and assessing debt-control strategies, particularly the potential effects of leadership and governance reforms on fiscal sustainability. The study also contributes to the application of epidemiological and compartmental modelling techniques in economics and finance by extending contagion-based approaches to the analysis of public debt. The resulting framework can support policymakers and other stakeholders in evaluating debt-control interventions and developing strategies for improved fiscal management and sustainability.

1.6 Contribution to Knowledge

The major contribution of this study is the development of a novel compartmental debt-contagion model that simultaneously incorporates Leadership Intelligence Quotient (LIQ) into the analysis of financial debt transmission and control in Nigeria. By integrating these dimensions within a unified mathematical framework, the study extends the application of compartmental and contagion based modelling from epidemiology and financial systems to public debt analysis and provides a basis for examining mechanisms of debt accumulation and control.

Literature review

2.1 Public Debt and Debt Sustainability

Public debt is an important instrument for financing government expenditure and development, but excessive debt can weaken repayment capacity, constrain investment, and threaten economic stability. Krugman (1988) associated excessive indebtedness with the debt-overhang effect, while Reinhart and Rogoff (2010) reported an association between high debt levels, slower economic growth, and increased fiscal vulnerability. The International Monetary Fund (2023) further emphasizes in its fiscal monitor report that sustainable debt management requires the capacity to meet present and future debt obligations without compromising economic development.

2.2 Mathematical and Compartmental Modelling of Debt Dynamics

Mathematical modelling provides a framework for analysing debt behaviour, forecasting trajectories, and evaluating debt-control policies. Samonas (2015) emphasized the value of financial modelling for economic planning and policy decisions. Compartmental models, originally developed for epidemiological systems, provide a useful framework for representing interacting components and transmission processes. Gai and Kapadia (2010) demonstrated the application of contagion principles to financial systems and systemic risk. Building on this perspective, Ogofotha and Ogwumu (2022) applied epidemiological modelling concepts to financial debt dynamics, demonstrating that debt accumulation can exhibit contagion like behaviour. However, their framework did not incorporate LIQ.

2.3 Leadership Intelligence, Governance and Debt Sustainability

Leadership and governance are important institutional dimensions of economic and fiscal performance. Simonton (2006) proposed approaches for assessing leadership cognitive ability and emphasized its relevance to policy effectiveness and national performance, while Winter (2005) highlighted the role of leadership traits in political decision-making and governance outcomes. Governance encompasses dimensions including transparency, accountability, government effectiveness, rule of law, and control of corruption. Kaufmann, Kraay, and Mastruzzi (2010) developed the Worldwide Governance Indicators (WGI), which provide widely used measures of governance performance. These perspectives suggest that leadership capacity and governance quality can influence fiscal discipline, economic productivity, and vulnerability to debt accumulation.

2.4 Theoretical and Empirical Framework

This study is anchored on Systems Theory, which views economic, political, and institutional components as interconnected elements of a dynamic system. Accordingly, debt accumulation is considered to result from interactions among economic productivity, governance quality, leadership intelligence, and borrowing behaviour. Existing studies have largely examined debt sustainability, financial contagion, governance, leadership effectiveness, and financial forecasting as separate areas. Although these studies provide important foundations, limited attention has been given to integrating these dimensions within a single mathematical framework.

Kaufmann et al. (2010) established a comprehensive governance measurement framework, while Samonas (2015) demonstrated the relevance of mathematical and statistical modelling to financial forecasting. Ogofotha and Ogwumu (2022) further demonstrated the feasibility of applying epidemiological modelling to financial debt dynamics. However, these approaches do not simultaneously incorporate Leadership Intelligence Quotient (LIQ) and compartmental debt dynamics.

2.5 Research Gap

The reviewed literature establishes important relationships among debt sustainability, economic performance, financial contagion, governance, leadership, and mathematical forecasting. Nevertheless, the literature reviewed in this study does not provide a unified compartmental framework that simultaneously incorporates LIQ and debt transmission dynamics for analysing and controlling financial debt crises in Nigeria. This gap provides the foundation for the present study, which extends the debt contagion approach by integrating these dimensions into a nonlinear mathematical model.

Model formulation

3.1 Model motivation

The proposed model is motivated by the limitations of conventional debt models, which primarily focus on macroeconomic variables such as GDP growth, inflation, exchange rates, and government expenditure, while giving limited attention to the effects of governance quality and leadership effectiveness on debt accumulation and management. Moreover, conventional econometric approaches may not adequately represent the dynamic transmission and interactions among economic productivity, governance structures, leadership quality, and debt propagation within an economy.

Previous studies have examined debt sustainability (Reinhart & Rogoff, 2010), financial contagion (Gai & Kapadia, 2010), governance effectiveness (Kaufmann et al., 2010), and leadership performance (Simonton, 2006) largely as independent dimensions. The limited integration of these factors into a unified mathematical framework creates a gap in understanding how governance quality and leadership intelligence influence debt accumulation and control.

Accordingly, this study extends the debt - contagion modelling approach by incorporating Leadership Intelligence Quotient (LIQ) as important determinants of debt transmission and control. The resulting framework is formulated as a system of nonlinear ordinary differential equations describing the interactions among people, economic activity, debt, and leadership quality. This formulation

provides the mathematical foundation for analysing the dynamics, equilibrium, stability, and control of financial debt in Nigeria.

3.2 Model Assumptions

The model is developed under the following assumptions:

i. People interact with the economy and influence debt accumulation.

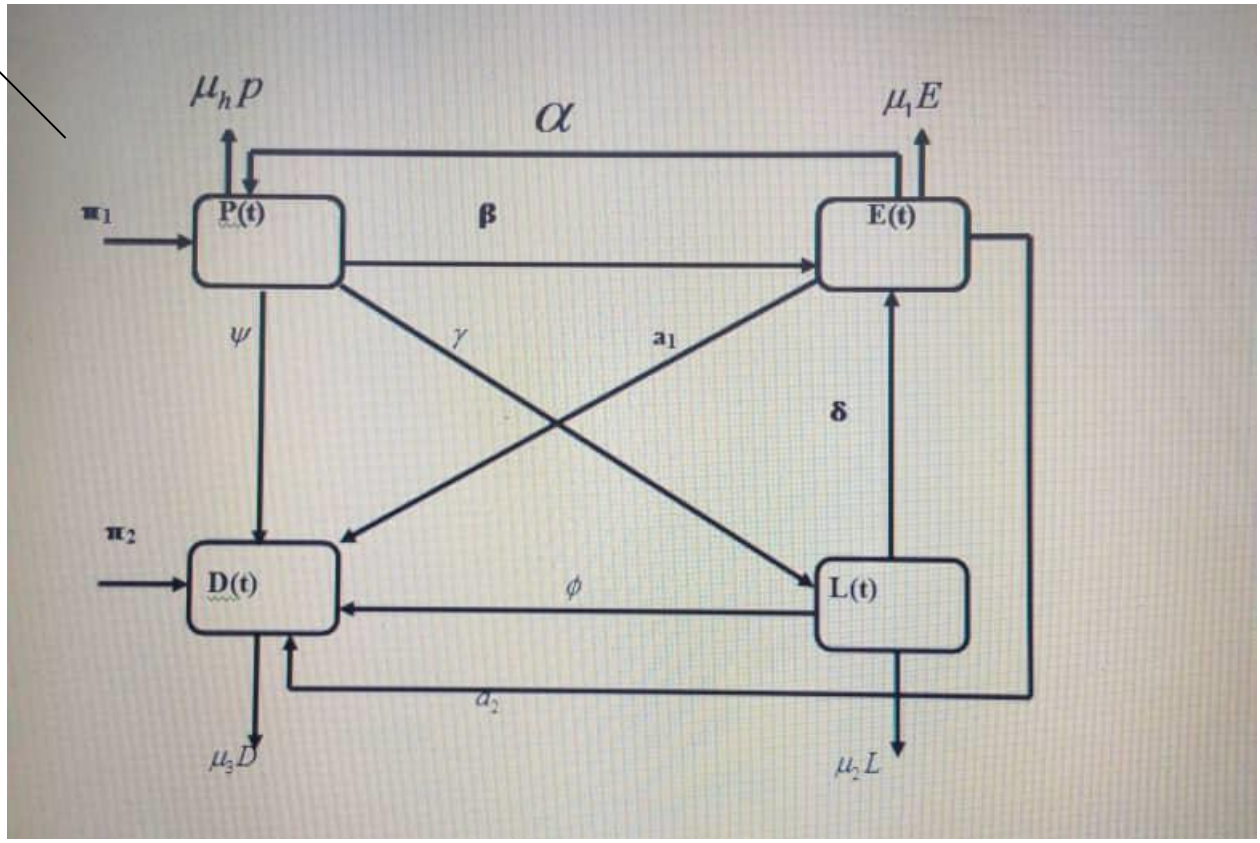
ii. Debt spreads through economic inefficiencies.

iii. Leadership quality affects debt-control effectiveness.

iv. Governance quality influences borrowing discipline.

3.3 Flow Diagram

The schematic representation of model Dynamics is presented below



3.4 Model Equations

Population Growth Equation

$$\frac{dP}{dt} = \pi_1 - \gamma PL - \beta P - \psi PD + \alpha P - \mu_h P \quad (3.1)$$

Economic Growth Equation

$$\frac{dE}{dt} = \beta P - a_2 D + \delta L - (a_1 + \alpha)E - \mu_1 E \quad (3.2)$$

Debt Growth equation

$$\frac{dD}{dt} = \pi_2 + \psi PD + \phi L + a_1 E + a_2 D - \mu_2 D \quad (3.3)$$

Leadership IQ Equation

$$\frac{dL}{dt} = -\phi L + \gamma PL - \delta L - \mu_3 L \quad (3.4)$$

3.5 Model Parameters

Table 3.1 Variable and Parameter Model

VARIABLE/ PARAMETER	DESCRIPTION
P(t)	The People or population
E(t)	The economics of a Nation
D(t)	The debt of a Nation
L(t)	Leadership Intelligence Quotient
π_1	Recruitment of the people
β	Impact of the people on the economic
γ	Corruption effect Rate of The people on Leadership
δ	Leadership effect on the Economics
a_1	Impact of Economics Boost on the debt crisis
a_2	Profit From Debt Management
ϕ	Positive affect of leadership on debt
ψ	Productivity Perspective Loss By the Leader
μ_h	Death of Humans causing Debt Crisis
μ_1	Economics Decline Rate Due to Debt Crisis
μ_2	Leadership IQ Decay Rate
α	Rate at which the people take from the economics without replacement
π_2	External borrowing rate
μ_3	Debt Decay

3.6 Existence and Uniqueness of Solution

$$\pi_1 - \gamma PL - \beta P - \psi PD + \alpha E - \mu_h P = 0 \to f_1$$
$$\beta P - a_2 D + \delta L - (a_1 + \alpha)E - \mu_1 E = 0 \to f_2$$
$$\pi_2 + \psi PD + \phi L + a_1 E + a_2 D - \mu_2 D = 0 \to f_3$$
$$-\phi L + \gamma PL - \delta L - \mu_3 L = 0 \to f_4$$

Computing the Jacobian Matrix of the solution.

$$J = \begin{bmatrix} \frac{\partial \text{Eqn1}}{\partial P} & \frac{\partial \text{Eqn1}}{\partial E} & \frac{\partial \text{Eqn1}}{\partial D} & \frac{\partial \text{Eqn1}}{\partial L} \\ \frac{\partial \text{Eqn2}}{\partial P} & \frac{\partial \text{Eqn2}}{\partial E} & \frac{\partial \text{Eqn2}}{\partial D} & \frac{\partial \text{Eqn2}}{\partial L} \\ \frac{\partial \text{Eqn3}}{\partial P} & \frac{\partial \text{Eqn3}}{\partial E} & \frac{\partial \text{Eqn3}}{\partial D} & \frac{\partial \text{Eqn3}}{\partial L} \\ \frac{\partial \text{Eqn4}}{\partial P} & \frac{\partial \text{Eqn4}}{\partial E} & \frac{\partial \text{Eqn4}}{\partial D} & \frac{\partial \text{Eqn4}}{\partial L} \end{bmatrix}$$

$$J=\begin{bmatrix} -\gamma L-\beta-\psi D-\mu_h & \alpha & -\psi P & -\gamma P \\ \beta & -(a_1+\alpha+\mu_1) & a_2 & \delta \\ \psi D & a_1 & \psi P-a_2-\mu_2 & \phi \\ \gamma L & 0 & 0 & \gamma P-\phi-\delta-\mu_3 \end{bmatrix}$$

$$\det(J)=\begin{bmatrix} -\gamma L-\beta-\psi D-\mu_h & \alpha & -\psi P_0 & -\gamma P_0 \\ \beta & -(a_1+\alpha+\mu_1) & a_2 & \delta \\ \psi D & a_1 & \psi P_0-a_2-\mu_2 & \phi \\ \gamma L & 0 & 0 & \gamma P_0-\phi-\delta-\mu_3 \end{bmatrix}$$

If det (J) ≠ 0 at a solution (P, E, D and L), then the Implicit Function Theorem guarantees a locally unique solution.

3.7 Positivity and Boundedness

Positivity means that all state variables of the model remain non-negative for all time $t>o.$

Consider the compartmental financial debt dynamics model with initial conditions

This means

$P(t)\geq o$, $E(t)\geq o$, $D(t)\geq o$, $L(t)\geq o$

Then, the solution

P(t), E(t), D(t), L(t) remain non-negative for every $t\geq 0$.

This shows that;

$P(\mathrm{t})\geq 0,\ E(\mathrm{t})\geq 0, D(\mathrm{t})\geq 0, \mathrm{L}(\mathrm{t})\geq 0 \forall t\geq 0.$

Proof.

$$\frac{dP}{dt}=\pi_1-\gamma PL-\beta P-\psi PD+\alpha E-\mu_h P \qquad (3.5)$$

$$\frac{dP}{dt}\geq -K_1 P,$$

$$P(t)\geq P(0)\ell^{-K_1 t}$$

$$P(t)>0, \forall t\geq 0.$$

$$\frac{dE}{dt}=\beta P-a_2 D+\delta L-(a_1+\alpha)E-\mu_1 E \qquad (3.6)$$

$$\frac{dE}{dt}\geq -K_2 E,$$

$$E(t)\geq E(0)\ell^{-K_2 t}>0$$

$$\frac{dD}{dt}=\pi_2+\psi PD+\phi L+a_1 E+a_2 D-\mu_2 D \qquad (3.7)$$

$$\frac{dD}{dt}\geq -K_3 D,$$

And

$$D(t)\geq D(0)\ell^{-K_3 t}\geq 0.$$

$$\frac{dL}{dt}=-\phi L+\gamma PL-\delta L-\mu_3 L \qquad (3.8)$$

$$\frac{dL}{dt} \geq -K_4 L$$

$$L(t) \geq L(0)\ell^{-K_4 t} > 0$$

(P(t), E(t), D(t),L(t) $\in \square_t^4$

Therefore, the positivity property confirms that the model is economically meaningful because all state variables remain non-negative throughout the period of analysis.

Hence the model provides realistic projections of debt dynamics and forms a reliable basis for evaluating fiscal policies aimed at achieving sustainable debt management in Nigeria.

3.8 Boundedness of Solutions

N(t) = P(t) + E(t) + D(t) +L(t)

Differentiate with respect to each term

$$\frac{dN}{dt} = \frac{dP}{dt} + \frac{dE}{dt} + \frac{dD}{dt} + \frac{dL}{dt} \tag{3.9}$$

Substituting equation (3.1 - 3.4) into equation (20) gives

$$\frac{dN}{dt} = (\pi_1 - \gamma PL - \beta P - \psi PD + \alpha E - \mu_h P) + \beta P - a_2 D + \delta L - (a_1 + \alpha)E - \mu_1 E)$$

$$+\pi_2 + \psi PD + \phi L + a_1 E + a_2 D - \mu_2 D) + -\phi L + \gamma PL - \delta L - \mu_3 L). \tag{3.10}$$

Collecting like terms from equation (3.10)

Therefore,

$$\frac{dN}{dt} = \pi_1 + \pi_2 - \mu_h P - \mu_1 E - \mu_2 D - \mu_3 L \tag{3.11}$$

Now let

$$\mu = \min(\mu_h, \mu_1, \mu_2, \mu_3),$$

Since

$$\mu_h P + \mu_1 E + \mu_2 D + \mu_3 L \geq \mu(P + E + D + L) = \mu N,$$

We obtain

$$\frac{dN}{dt} \leq (\pi_1 + \pi_2) - \mu N.$$

Let $\Pi = \pi_1 + \pi_2$

Hence,

$$\frac{dN}{dt} \leq \Pi - \mu N \tag{3.12}$$

From equation (3.12), applying the integrating factor method

$$\frac{dN}{dt} + \mu N \leq \Pi$$

Multiplying by $\ell^{\mu t}$

$$\frac{d}{dt}(N\ell^{\mu t}) \leq \Pi\ell^{\mu t} \tag{3.13}$$

Integrating equation (3.13) from 0 to t.

$$N(t)\ell^{\mu t} - N(0) \leq \Pi \int_0^t \ell^{\mu s} ds$$

Therefore,

$$N(t) \le N(0)\ell^{-\mu t} + \frac{\Pi}{\mu}(1-\ell^{-\mu t}).$$

Taking the limit as $t \to \infty$,

$$\frac{\limsup N(t)}{t \to \infty} \le \frac{\Pi}{\mu} = \frac{\pi_1 + \pi_2}{\mu}$$

Hence, the feasible region is

$$\Pi = \left\{ (P,E,D,L) \in \square_t^4 : P + E + D + L \le \frac{\pi_1 + \pi_2}{\mu} \right\}$$

3.9 Feasible Region

The feasible region of the model is

The positivity of solutions guarantees that all state variables remain non-negative for all time, ensuring the economic relevance of the model. Furthermore, boundedness analysis demonstrates that the total system population remains within a finite region, implying that debt accumulation, economic productivity and leadership dynamics participation evolve within realistic economic limits. Consequently, the model is mathematically well-posed and economically feasible.

3.10 Debt Free Equilibrium (DFE)

The Debt-Free Equilibrium (DFE) is obtained by setting the derivatives in equations (3.1) – (3.4) equal to zero and imposing D = 0 and L = 0. The resulting steady-state equations for the People and Economy compartments are solved simultaneously to obtain the equilibrium values P^0 and E^0.

Thus, Debt - Free Equilibrium (DFE) is given by

$$(P_0, E_0, D_0, L_0)$$

$$= \left(\frac{\pi_1 - \pi_2 + (\alpha - a_1)\dfrac{\beta\pi_1 - (2\beta - \mu_h)\pi_2}{(\beta - \mu_h)(2a_1 + \alpha + \mu_1) - \beta(\alpha - a_1)}}{\beta + \mu_h}, \frac{\beta\pi_1 - (2\beta - \mu_h)\pi_2}{(\beta - \mu_h)(2a_1 + \alpha + \mu_1) - \beta(\alpha - a_1)}, 0, 0 \right)$$

where P_0 and E_0 are the equilibrium values obtained from the steady - state equations and the expressions depend on the model parameters.

The resulting equilibrium represents the stable financial state in which public debt is absent (D = 0) and the leadership/governance-related debt component is at its debt-free equilibrium level (L= 0). In particular, the model indicates that debt can only be eliminated when the external financial shock parameter satisfies ($\pi_2 = 0$)

This equilibrium provides the reference state for the subsequent derivation of the Basic Debt Reproduction Number and the local stability analysis.

3.11 Basic Debt Reproduction Number (R_o)

According to Van den Driessche and Watmough (2002), the basic reproduction number provides a threshold for determining whether a dynamic process will persist or disappear. Brauer and Castillo-Chavez (2012) similarly identified R_0 as an important indicator for assessing intervention strategies in compartmental models. In the present study, R_0 represents the expected number of secondary debt-generating events arising from an existing debt source during its effective period within the Nigerian economy.

The Basic Debt Reproduction Number is obtained using the next-generation matrix method. The debt-transmitting components of the model are used to construct the transmission matrix F, while the

transition matrix V represents the natural removal of financial distress through debt repayment, governance reforms, and economic stabilization.

The transmission matrix is given by;

From the general equation differentiating the debt transmitting parameter i.e. equation (3.3) and (3.4) we have

$$F = \begin{pmatrix} \psi \dfrac{\pi_1 - \pi_2 + (\alpha - a_1)\dfrac{\beta\pi_1 - (2\beta - \mu_h)\pi_2}{(\beta - \mu_h)(2a_1 + \alpha + \mu_1) - \beta(\alpha - a_1}}{\beta + \mu_h} & 0 \\ 0 & \gamma \dfrac{\pi_1 - \pi_2 + (\alpha - a_1)\dfrac{\beta\pi_1 - (2\beta - \mu_h)\pi_2}{(\beta - \mu_h)(2a_1 + \alpha + \mu_1) - \beta(\alpha - a_1)}}{\beta + \mu_h} \end{pmatrix}$$

$$\mathrm{V} = \begin{pmatrix} a_2 + \mu_2 & -\phi \\ 0 & \delta + \mu_3 + \phi \end{pmatrix}$$

The Basic Reproduction Number (R_0) is the largest eigenvalue (spectral radius) of the matrix FV^{-1}

$$R_0 = \rho FV^{-1}$$

The inverse of a 2x2 matrix is given by

$$V^{-1} = \frac{1}{\det(V)}\begin{pmatrix} \upsilon_{22} & -\upsilon_{12} \\ -\upsilon_{21} & \upsilon_{11} \end{pmatrix},$$

The determinant is

$$\det(V) = (a_2 + \mu_2)(\delta + \mu_3 + \phi) - (0)(-\phi)$$

Thus the inverse is

$$V^{-1} = \frac{1}{(a_2 + \mu_2)(\delta + \mu_3 + \phi)}\begin{bmatrix} \delta + \mu_3 + \phi & \phi \\ 0 & a_2 + \mu_2 \end{bmatrix}$$

$$FV^{-1} = \begin{pmatrix} \psi \dfrac{\pi_1 - \pi_2 + (\alpha - a_1)\dfrac{\beta\pi_1 - (2\beta - \mu_h)\pi_2}{(\beta - \mu_h)(2a_1 + \alpha + \mu_1) - \beta(\alpha - a_1}}{\beta + \mu_h} \cdot \dfrac{\delta + \mu_3 + \phi}{(a_2 + \mu_2)(\delta + \mu_3 + \phi)} & 0 \\ \psi \dfrac{\pi_1 - \pi_2 + (\alpha - a_1)\dfrac{\beta\pi_1 - (2\beta + \mu_h)\pi_2}{(\beta - \mu_h)(2a_1 + \alpha + \mu_1) - \beta(\alpha - a_1}}{\beta + \mu_h} \cdot \dfrac{\phi}{(a_2 + \mu_2)(\delta + \mu_3 + \phi)} & \gamma \dfrac{(\pi_1 - \pi_2 + (\alpha - a_1)\dfrac{\beta\pi_1 - (2\beta - \mu_h)\pi_2}{(\beta - \mu_h)(2a_1 + \alpha + \mu_1) - \beta(\alpha - a_1)} \cdot \dfrac{a_2 + \mu_2}{(a_2 + \mu_2)(\delta + \mu_3 + \phi)}}{\beta + \mu_h} \end{pmatrix}$$

Simplifying the eigenvalue FV^{-1} are the solution to;

$$\left(\psi \frac{\pi_1 - \pi_2 + (\alpha - a_1)\dfrac{\beta\pi_1 - (2\beta - \mu_h)\pi_2}{(\beta - \mu_h)(2a_1 + \alpha + \mu_1) - \beta(\alpha - a_1} \cdot \dfrac{\delta + \mu_3 + \phi}{(a_2 + \mu_2)(\delta + \mu_3 + \phi)}}{\beta + \mu_h}\right) \cdot \left(\gamma \frac{(\pi_1 - \pi_2 + (\alpha - a_1)\dfrac{\beta\pi_1 - (2\beta - \mu_h)\pi_2}{(\beta - \mu_h)(2a_1 + \alpha + \mu_1) - \beta(\alpha - a_1)} \cdot \dfrac{a_2 + \mu_2}{(a_2 + \mu_2)(\delta + \mu_3 + \phi)}}{\beta + \mu_h}\right) = 0$$

The basic reproduction number R_o is,

$$R_O = \frac{(\psi(\delta + \mu_3) + \gamma(a_2 + \mu_2))(2\pi_1 a_1 + \pi_1\mu_1 - \pi_2 a_1 - 2\pi_2\alpha - \pi_2\mu_2)}{((a_2 + \mu_2)(\delta + \mu_3) - \phi^2)(3a_1\mu_h + 2a_1\mu_h + \alpha\mu_h + \beta\mu_1 + \mu_1\mu_h))} \quad (3.23)$$

3.12 Sensitivity Analysis

The objective of this subsection is to use partial differentiation to obtain the sensitivity indices of our model parameters. The sensitivity Index measures how changes in modal parameters affect R_o , this helps identify the most influential parameters in financial debt control in Nigeria.

The normalized sensitivity index for a parameter X is given by;

$$S_X = \frac{\partial R_0}{\partial X} * \frac{X}{R_0} \tag{3.24}$$

A positive sensitivity index indicates that an increase in the corresponding parameter increases R_0, whereas a negative index indicates an inverse relationship.

The sensitivity indices obtained for the model parameters are presented in Table 3.1.

Table 3.2 Sensitivity Analysis Table

S/n	Parameter	Sensitivity Index	Status
1	β	-0.6815084052	Strong Negative
2	δ	0.1555343511	Positive
3	ϕ	-2.061068702	Strongest Negative Influence
4	a_1	-0.8859609269	Stronger Negative
5	μ_h	-0.3184915946	Negative
6	a_2	0.7546119590	Positive
7	μ_1	-0.0151437908	Negative
8	μ_2	0.1509223919	Positive
9	α	0.2859477127	Positive
10	π_1	-0.3594771242	Strong Negative
11	π_2	2.653465347	Strongest Positive Influence
12	ψ	0.1250000000	Positive
13	μ_3	0	0
14	γ	0.875000000	Positive

3.13 Local Stability (Jacobian Matrix Eigen values)

The local stability can be determined by Jacobian matrix , that is, by finding the eigenvalues of the Jacobian matrix. If all the eigenvalues are negative or have negative real part then the system will be locally asymptotically stable. That is, financial crisis does not spread but rather diminishe. Otherwise unstable, i.e. public debt continues to spread and increases.

In this study, the local stability analysis determines whether the Nigerian economy returns to the Debt-Free Equilibrium following small disturbances in public debt level

Therefore, the Jacobian of the system is,

$$J = \begin{bmatrix} -\gamma L - \beta - \psi D + \alpha - \mu_h & 0 & -\psi P & -\gamma P \\ \beta & -(a_1 + \alpha) - \mu_1 & a_2 & \delta \\ \psi D & a_1 & \psi P - a_2 - \mu_2 & \phi \\ \gamma L & 0 & 0 & \gamma P - \phi - \delta - \mu_3 \end{bmatrix} \tag{3.25}$$

Evaluating (3.25) at Debt Free Equilibrium gives

$$J_{DFE}=\begin{bmatrix} -\gamma L-\beta-\psi D+\alpha-\mu_h & 0 & -\psi P & -\gamma P \\ \beta & -(a_1+\alpha)-\mu_1 & a_2 & \delta \\ \psi D & a_1 & \psi P-a_2-\mu_2 & \phi \\ \gamma L & 0 & 0 & \gamma P-\phi-\delta-\mu_3 \end{bmatrix} \tag{3.26}$$

$$\lambda_4=\gamma P_0-\phi-\delta-\mu_3$$

$$\gamma P_0-\phi-\delta-\mu_3 \angle 0$$

$$\frac{\pi_1-\pi_2+(\alpha-a_1)\dfrac{\beta\pi_1-(2\beta-\mu_h)\pi_2}{(\beta-\mu_h)(2a_1+\alpha+\mu_1)-\beta(\alpha-a_1)}}{\beta+\mu_h}<\frac{\gamma+\phi+\delta+\mu_3}{\gamma}$$

3.3 Parameter, Values, Interpretation and Sources Table

Parameter	Value	Interpretation	Source
β	0.03	Impact of the people on the economics (new borrowing as % of GDP).	IMF Country Reports, Nigeria's Debt Management Office (DMO) annual statements. (2021)
δ	0.003	Leadership Impact on the economics	Central Bank of Nigeria (CBN) macroeconomic data. (2019)
ϕ	0.45	Positive Impact of leadership on debt	World Bank Governance Indicators (WGI) or Transparency International CPI.(2010)
a_1	0.03	Impact of Economic productivity or boost on the Debt crisis.	National Bureau of Statistics (NBS) GDP reports. (2018)
μ_h	0.02	Death of human causing debt crisis	Household debt surveys (CBN or World Bank). (2021).
a_2	0.5	Profit from debt management	IMF Fiscal Monitor or Federal Ministry of Finance. (2022).
μ_1	0.01	Economic decline rate due to debt crisis.	Nigeria ministry of Financial (2017).
μ_2	0.1	Leadership IQ decay rate.	Nigeria Deposit Insurance Corporation, (NDIC). (2020).
α	0.0001	Rate at which the funds are withdraw from the economy without repayment	DMO, Nigeria. (2023).
π_1	0.015	Recruitment of the people	Organization of petroleum exporting counties.(2021)
π_2	0.03	External borrowing rate	Central bank of Nigeria. (2022)
ψ	0.03	Fiscal policy responsiveness to debt.	Federal republic of Nigeria.(2023)
μ_3	0.0	Debt decay (assumed negligible)	IMF. (2023).
γ	0.2	Corruption impact rate of the people on leaderships	Transparency International. (2023)

Results and Discussion

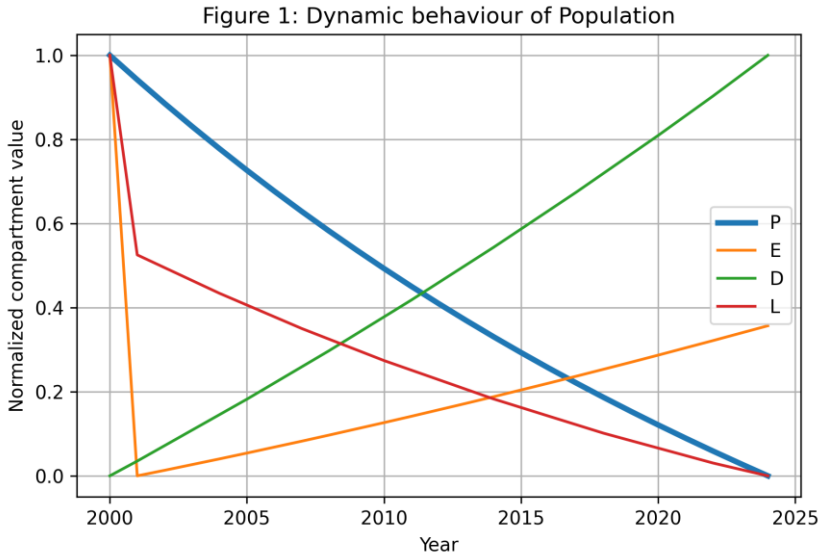
4.1 Numerical Simulation and Model Validation

Numerical simulations were performed using Maple to examine the interactions among People (P), Economy (E), Debt (D), and Leadership Intelligence Quotient (LIQ) under selected parameter variations. The simulations complement the analytical results established in Chapter Three and examine the effects of economic growth, LIQ, debt recruitment, and related intervention parameters on public debt dynamics.

The four state variables remained non-negative and bounded throughout the simulations, supporting the positivity and boundedness properties established analytically. The simulations further showed that improvements in economic performance, LIQ, debt repayment mechanisms, and prudent fiscal policies reduce debt persistence, whereas excessive borrowing and weak fiscal discipline promote debt accumulation. The agreement between the sensitivity results and the numerical trajectories, particularly for LIQ, debt recruitment, and fiscal policy effectiveness, supports the behaviour of the proposed model.

4.2 Numerical Simulation Results

4.2.1 Effect of Economic Growth on Public Debt Dynamics



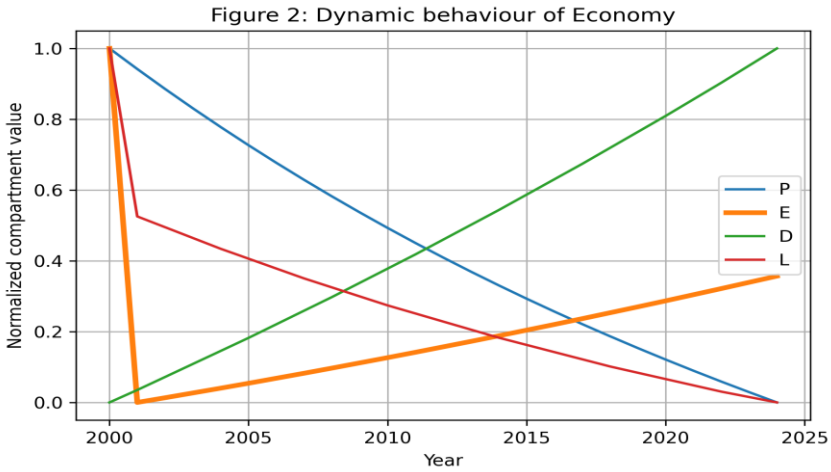
Interpretation

Figure 4.1: Numerical simulation showing the effect of economic growth on the People (P), Economy (E), Debt (D), and Leadership Intelligence Quotient (LIQ) compartments.

The simulation shows that improved economic performance increases Economy (E) while Debt (D) declines progressively. Stronger economic productivity and revenue generation reduce dependence on borrowing and improve debt-servicing capacity. The People (P) compartment remains stable, while improved LIQ reinforces the positive effects through better fiscal management.

4.2.2 Effect of Improved Economic Performance on Public Debt Dynamics

Figure 4.2: Numerical simulation illustrating the effect of improved economic performance on the dynamics of the People (P), Economy (E), Debt (D), and Leadership Intelligence Quotient (LIQ) compartments.



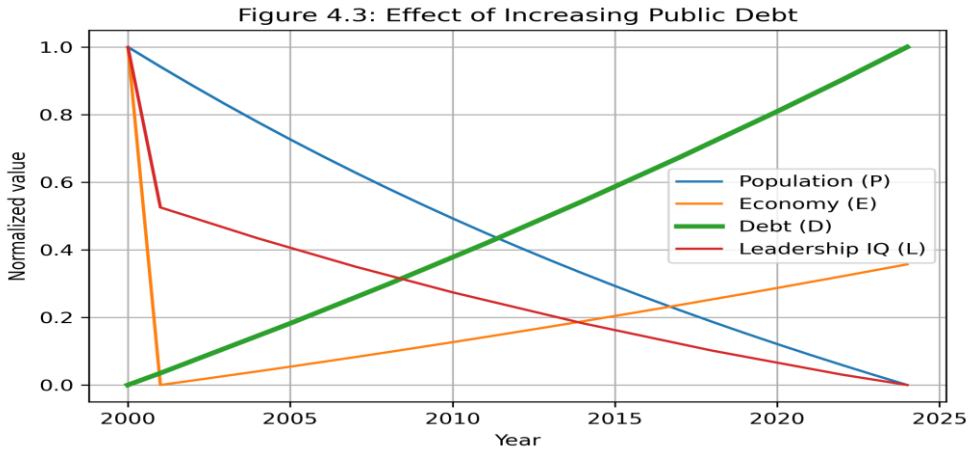
Interpretation

Figure 4.2: Numerical simulation illustrating the effect of improved economic performance on the P, E, D, and LIQ compartments.

Improved economic performance produces a sustained increase in E and a decline in D, indicating that economic expansion reduces borrowing requirements and debt persistence. The result supports the Basic Debt Reproduction Number analysis and the positivity and boundedness properties established in Chapter Three.

4.2.3 Combined Effect of Leadership Intelligence Quotient (LIQ) and Economic Growth on Public Debt Dynamics

Figure 4.3: Numerical simulation illustrating the combined effect of Leadership Intelligence Quotient (LIQ) and economic growth on the dynamics of the People (P), Economy (E), Debt (D), and Leadership Intelligence Quotient (LIQ) compartments.



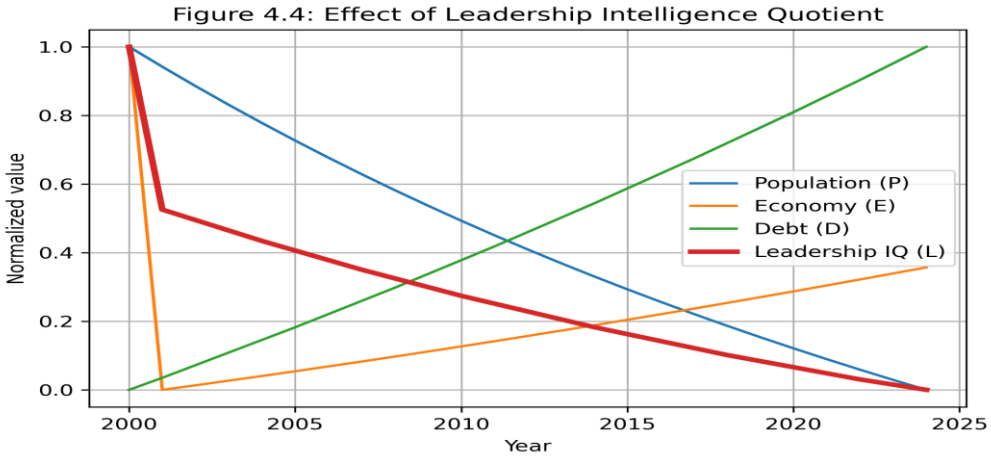
Interpretation

Figure 4.3: Numerical simulation illustrating the combined effect of LIQ and economic growth on the P, E, D, and LIQ compartments.

The combined improvement in LIQ and economic growth produces a more favourable debt trajectory than either intervention independently. Debt (D) declines while Economy (E) improves and People (P) remains stable, demonstrating the complementary influence of effective leadership and economic performance on debt sustainability.

4.2.4 Effect of the Debt Recruitment Rate (π_2) on Public Debt Dynamics

Figure 4.4: Numerical simulation illustrating the effect of variations in the debt recruitment rate (π_2) on the dynamics of the People (P), Economy (E), Debt (D), and Leadership Intelligence Quotient (LIQ) compartments.



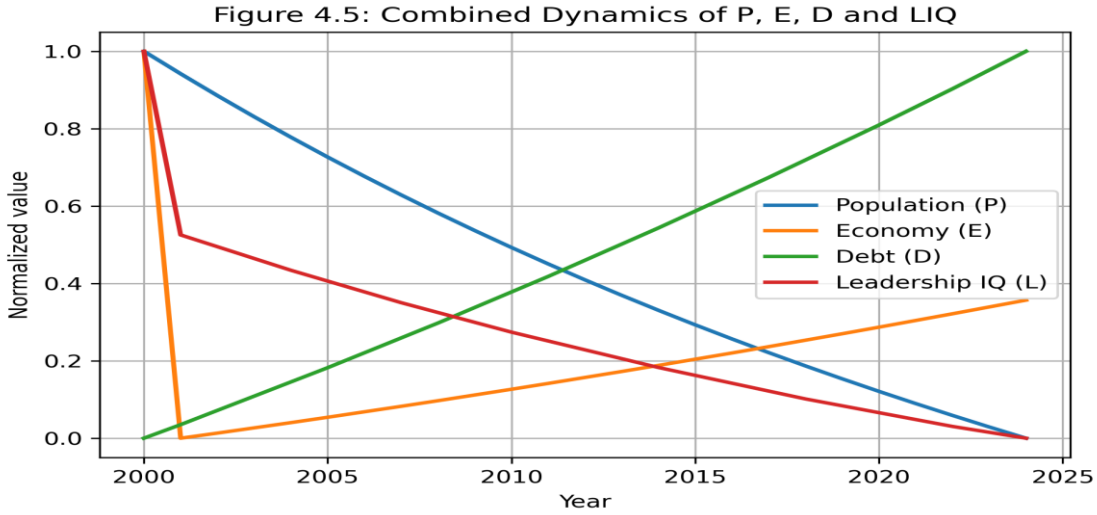
Interpretation

Figure 4.4: Numerical simulation illustrating the effect of variations in the debt recruitment rate on the P, E, D, and LIQ compartments.

An increase in the debt recruitment rate produces a corresponding increase in Debt (D) and weakens Economy (E) as greater resources are required for debt servicing. The result is consistent with the Basic Debt Reproduction Number and sensitivity analysis, which identify debt recruitment as an influential driver of debt persistence.

4.2.5 Effect of Leadership Intelligence Quotient (LIQ) on Public Debt Dynamics

Figure 4.5: Numerical simulation illustrating the effect of variations in the Leadership Intelligence Quotient (LIQ) parameter on the dynamics of the People (P), Economy (E), Debt (D), and Leadership Intelligence Quotient (LIQ) compartments.



Interpretation

Figure 4.5: Numerical simulation illustrating the effect of variations in LIQ on the P, E, D, and LIQ compartments.

Improved Leadership Intelligence Quotient (LIQ) produces a gradual reduction in Debt (D) and improvement in Economy (E), while People (P) remains positive and stable. The result demonstrates an inverse relationship between LIQ and debt accumulation and supports the analytical sensitivity findings.

4.3 Discussion of Findings

The numerical results demonstrate that the proposed compartmental financial debt dynamics model captures the interactions among People (P), Economy (E), Debt (D), and Leadership Intelligence

Quotient (LIQ) and supports the theoretical results established in Chapter Three. Economic growth and improved LIQ reduce debt accumulation, whereas increased debt recruitment promotes debt persistence.

The findings further demonstrate that LIQ and economic growth are complementary determinants of public debt sustainability, with their combined effect producing more favourable debt dynamics. The results are consistent with Brauer and Castillo-Chavez (2012) on coordinated interventions in compartmental systems and Chitnis, Hyman and Cushing (2008) on prioritising influential parameters identified through sensitivity analysis.

Conclusion and Recommendations

5.1 Conclusion

This study developed a compartmental mathematical model to investigate the dynamics and control of public debt in Nigeria. The model consists of four interacting compartments: People (P), Economy (E), Debt (D), and Leadership Intelligence Quotient (LIQ), with LIQ incorporated as a strategic intervention variable influencing fiscal discipline, economic productivity, and debt sustainability.

The mathematical analysis established the existence and uniqueness of solutions, positivity, boundedness, feasible region, Debt-Free Equilibrium (DFE), Basic Debt Reproduction Number, local stability, and sensitivity analysis. The reported Basic Debt Reproduction Number was $R_0 = 0.0667$, indicating that the public debt system remains asymptotically stable under the prevailing control conditions. Maple simulations supported the analytical results, showing that improvements in LIQ and economic performance reduce debt accumulation, whereas excessive borrowing and weak governance increase debt persistence. Sensitivity analysis identified LIQ and debt recruitment among the influential parameters affecting debt sustainability.

The study concludes that Nigeria's public debt dynamics can be effectively analysed using a compartmental mathematical framework integrating People, Economy, Debt, and Leadership Intelligence Quotient. The analytical and numerical results demonstrate that debt sustainability depends on the interaction of economic performance, borrowing behaviour, governance quality, and leadership effectiveness. Improved leadership effectiveness contributes to reduced debt accumulation, stronger economic performance, improved fiscal discipline, and enhanced institutional accountability, whereas poor leadership and excessive borrowing increase the likelihood of persistent debt growth and fiscal instability. The consistency between the theoretical analyses and Maple simulations supports the reliability of the proposed model.

5.2 Recommendations

Based on the findings, the Federal Government of Nigeria should promote responsible borrowing and ensure that public debt is directed towards productive investments capable of generating long-term economic returns. The Debt Management Office (DMO) should incorporate mathematical modelling into debt sustainability assessment, forecasting, and policy evaluation.

The Central Bank of Nigeria (CBN) should continue policies promoting macroeconomic and exchange-rate stability and inflation control, while the National Assembly should strengthen legislative oversight of public borrowing to ensure transparency and accountability. Leadership development institutions should promote continuous leadership training, ethical governance, evidence-based policymaking, and strategic public financial management to improve Leadership Intelligence Quotient within public institutions.

5.3 Contribution to Knowledge

This study make a significant contribution to the literature by developing a compartmental mathematical model for Nigeria's public debt dynamics and incorporating Leadership Intelligence Quotient as an endogenous variable affecting debt sustainability. It establishes the theoretical properties

of the model and demonstrates through numerical simulation that improvements in leadership effectiveness and economic performance reduce debt accumulation.

5.4 Suggestions for Further Research

Further studies may extend the proposed model by incorporating additional macroeconomic variables, applying the model to other developing economies, integrating stochastic processes, employing machine learning and artificial intelligence for parameter estimation, developing regional and state-level debt models within Nigeria, and examining the interaction between climate-related financial risks, governance quality, and public debt sustainability.

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