

Dynamic Modeling and Multi-Objective Energy Optimization of Nepalese Manufacturing Systems Using AI and Numerical Differential Equation Methods

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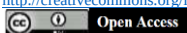
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ABSTRACT

Manufacturing plants in Nepal need operating decisions that lower energy cost without compromising delivery, yet few analytical tools link plant dynamics, electricity tariffs and optimisation in a transparent and reproducible way. This paper develops and tests such a workflow on a representative electrically heated batch-processing line. A six-state ordinary-differential-equation (ODE) model couples process temperature, order backlog, energy, cost, throughput and cumulative delay. Seven numerical integrators (Euler, Heun, RK4, RK45, DOP853, Radau and BDF) are verified against a high-accuracy reference; multilayer-perceptron, random-forest and gradient-boosting surrogates are trained on Latin-hypercube simulations; and NSGA-II is used to minimise specific energy consumption, energy cost per unit and a backlog-delay index. Fixed-step RK4 with a two-minute step reproduced reference objectives to within 3.4×10^{-5} relative error across 30 random designs, whereas a loose adaptive tolerance (10–3) produced temperature errors above 3 °C. A neural surrogate reached $R^2 \geq 0.998$ from 3,000 simulations. Setpoint temperature explained 76–89% of output variance (first-order Sobol indices). A compromise operating point cut energy per unit by 8.7% and cost per unit by 9.5% relative to a conventional baseline, and a “no-regret” point cut 8.1% and 9.1% with no delay penalty; energy and cost gains held in all 2,000 parameter-uncertainty draws. Surrogate-assisted search with 400 simulations did not outperform random sampling at equal budget. Because plant parameters are assumed rather than measured, the numerical values are illustrative; the contribution is the verified workflow and the trade-off structure it exposes.

Introduction

Industrial energy use sits at the centre of two policy conversations that are particularly pressing for developing economies: how to keep production competitive when energy is a significant operating cost, and how to reduce emissions without constraining growth. Recent assessments by the International Energy Agency continue to treat efficiency in industry as one of the most cost-effective levers available to policy makers [1], and management-system standards such as ISO 50001 formalise how organisations should establish energy baselines, performance indicators and improvement targets [2]. Both framings depend on a credible link between how a plant is operated and how much energy it consumes per unit of output. That link is dynamic: furnaces, dryers and ovens have thermal inertia, production queues build and drain over a shift, and electricity prices change with the time of day.

Nepal adds distinctive features to this picture. Domestic electricity generation is dominated by hydropower, much of it run-of-river, so supply conditions and their economics vary strongly with the season [3], and the country’s climate commitments identify the energy sector as a priority for low-carbon development [4]. In small and medium plants with limited instrumentation, setpoints and shift patterns are frequently chosen by experience rather than by quantitative analysis. That premise motivates this

work, although it is not itself tested here. Where hydropower-based electricity is available, electrification of process heat and tariff-aware scheduling may offer a route to lower cost and lower emissions, but plant managers need decision-support tools that are transparent, inexpensive to run and honest about uncertainty.

Research on manufacturing energy has long shown that specific energy consumption depends strongly on operating rate, because machines draw substantial power even when they are idle or lightly loaded [5], [6]. Capturing such behaviour requires models that follow the state of the plant through time. Ordinary differential equations (ODEs) are the natural language for lumped dynamics of this kind, but their use inside energy-optimisation studies raises a practical question that deserves explicit treatment: which numerical integration scheme is accurate and economical enough when the model is evaluated thousands of times by an optimiser? The question is not trivial. Heater controllers saturate, shift schedules change the production rate abruptly, and tariff boundaries introduce discontinuities, and these non-smooth features degrade the textbook convergence orders of high-order Runge–Kutta methods [7], [8]. At the same time, machine-learning surrogates [9] and evolutionary multi-objective search [10] promise to make optimisation tractable, yet their benefit relative to simple baselines should be demonstrated rather than assumed.

This paper responds with a computational case study of a representative electrically heated batch-processing line, organised around three research questions:

1. **RQ1.** How accurate and economical are common explicit and implicit ODE solvers for a non-smooth plant model with controller saturation and piecewise-constant operating schedules?
2. **RQ2.** How accurately can data-driven surrogates trained on a modest number of simulations reproduce the energy, cost and delay objectives, and does surrogate assistance improve multi-objective search at equal simulation budget?
3. **RQ3.** Which operating variables dominate the objectives, and how robust are the resulting trade-offs to uncertainty in physical parameters?

The contributions are as follows. First, a compact six-state ODE model is formulated that couples thermal dynamics, order backlog, electricity cost and delay. Second, seven integrators are verified on that model against a high-accuracy reference, and the observed convergence orders are reported rather than assumed. Third, an equal-budget comparison of direct, surrogate-assisted and random search is presented. Fourth, global sensitivity analysis and paired Monte Carlo robustness tests are used to qualify the trade-offs. Fifth, the limits of an assumed-parameter study are stated explicitly.

A framing note is essential. No plant measurements were available to the authors, so the model parameters in Table II are assumed values chosen to be physically plausible for a mid-sized thermal line. Every number reported below is therefore an output of that model, not an observation of a Nepalese factory, and absolute values should be read as illustrative. What transfers to practice is the workflow, the verified numerical choices, and the structure of the trade-offs. Section II reviews related work and identifies the research gap, Section III describes the methodology, Section IV presents the results, Section V discusses their meaning, and Sections VI–VIII cover limitations, future scope and conclusions.

LITERATURE REVIEW

A. Energy Behaviour of Manufacturing Systems

Gutowski et al. [5] analysed the electrical energy requirements of a range of manufacturing processes and reported that process power consists of a large, largely fixed component and a smaller component that scales with the amount of material processed. The practical consequence is that specific energy falls as the processing rate rises, until other limits intervene. Duflou et al. [6] extended the discussion to a systems view, organising efficiency measures across the unit process, the production line and the factory, and emphasising that interactions between levels, such as the effect of scheduling on idle time, matter as much as machine-level improvements. These contributions remain foundational, but they are largely static or steady-state in character. They describe average power and typical operating points rather than the transient interplay of thermal inertia, queues and price signals that determines the energy cost of an actual shift. ISO 50001 [2] institutionalises continual improvement through baselines

and performance indicators, yet it deliberately does not prescribe how to compute the best operating point, which leaves a methodological space for model-based analysis.

Industry-level assessments such as those of the IEA [1] describe the policy importance of efficiency but do not resolve plant-level questions. For Nepal, utility reporting such as the annual review of the Nepal Electricity Authority [3] documents supply and system conditions rather than the dynamics of individual factories. Plant-level modelling for small and medium manufacturers therefore has to be developed locally and has to be cheap enough to be used.

B. Dynamic Modelling and Numerical Integration

Lumped-parameter ODE models are a standard compromise between fidelity and tractability. Fixed-step explicit methods such as Euler, Heun and the classical fourth-order Runge–Kutta (RK4) scheme are simple to implement and, on vectorised hardware, can advance an entire population of candidate designs simultaneously. Embedded pairs such as Dormand–Prince [11] adapt the step to a local error estimate and are the default in many software environments [12], [13]. Implicit methods such as Radau and backward differentiation formulae are designed for stiff systems in which fast and slow time scales coexist [14]. Theory predicts convergence of order p for a method of order p only when the right-hand side is sufficiently smooth [7], [8]. Plant models are rarely smooth: actuator limits, switching schedules and thresholds introduce kinks and discontinuities, and an analyst who has not verified the solver may report energy figures that are partly numerical artefacts. In optimisation, where thousands of candidates are evaluated, an under-resolved solver can also distort the objective landscape and mislead the search.

C. AI and Scientific Machine Learning

Machine learning enters energy modelling in two distinct ways. The first uses data-driven models as surrogates for expensive simulators or experiments. Multilayer perceptrons, which are universal approximators [15] and are commonly trained with stochastic gradient methods such as Adam [16], random forests [17] and gradient-boosted trees [18] are frequent choices. The second way embeds or discovers differential equations inside learning models. Physics-informed neural networks add equation residuals to the training loss [19]–[21], sparse identification of nonlinear dynamics (SINDy) recovers parsimonious governing equations from time series [22], and neural ODEs parameterise vector fields with networks and rely on adaptive solvers in their forward and backward passes [23].

These methods are attractive when the physics is partly unknown and measured data are plentiful. In small plants with sparse instrumentation, an explicit physics-based model with a few interpretable parameters is easier to audit and calibrate, and machine learning is most defensible where it accelerates computation rather than replaces understanding. Model predictive control [24] offers a route from dynamic models to real-time operation, but it presupposes a validated model and online computing infrastructure that many small plants do not have.

D. Multi-Objective Optimisation and Surrogate Assistance

Operating a plant involves conflicting goals such as energy, cost, timeliness and quality, so a single optimum rarely exists. Multi-objective optimisation seeks the set of non-dominated solutions that exposes the trade-offs [25]. Evolutionary algorithms suit nonlinear, simulation-based problems. NSGA-II [10] combines fast non-dominated sorting with crowding-distance diversity preservation, while reference-point and decomposition-based methods such as NSGA-III [26] and MOEA/D [27] extend the toolbox to many-objective settings, and open-source frameworks such as pymoo [28] make these algorithms widely accessible. The quality of an approximation to the Pareto front is commonly summarised by hypervolume, the measure of objective space dominated by the front relative to a reference point [29]. Surrogate-assisted evolutionary computation replaces some or all expensive evaluations with approximate models [9], and Gaussian-process-based efficient global optimisation is the classical example [30]. The approach pays off when simulation is costly relative to model training and when the surrogate is accurate where it matters, near the Pareto front. Whether those conditions hold is an empirical question for each application.

Sahani et al. [35] investigated the mechanical and operational behavior of steel manufacturing processes. Their subsequent studies explored mathematical modeling of woolen knitting dynamics and

plastic pipe performance using Laplace transforms and fourth-order Runge–Kutta methods [36], [37]. In structural engineering, Sahani [38], [39] examined AI-enhanced finite element analysis and IoT-based structural health monitoring, while Sahani et al. [40] investigated machine learning approaches for predicting numerical integration errors. Further studies addressed heat exchanger optimization using numerical methods and genetic algorithms [41], industrial plant performance assessment [42], and mathematical modeling of cement manufacturing systems [43]. Research also examined maintenance optimization and jaw crusher performance in cement plants [44], [45], along with reliability-centered maintenance and statistical reliability modeling of industrial equipment [46], [47]. Additionally, numerical methods were applied to vibration analysis and heat conduction problems in mechanical engineering [48], [49]. Finally, Sahani et al. [50] assessed the reliability of a plywood production facility using Laplace transforms and fourth-order Runge–Kutta methods. Collectively, these studies demonstrate the value of mathematical modeling, numerical simulation, artificial intelligence, and optimization in improving industrial performance and reliability. However, integrating these techniques with energy and economic optimization under uncertain operating conditions remains an important research opportunity.

E. Synthesis and Research Gap

Table I summarises representative studies, what each contributes, and the gap relative to the present work. Three gaps follow. First, there is a need for an integrated and reproducible workflow that combines dynamics, tariffs and multi-objective search for small plants, rather than treating them separately. Second, solver choice in energy-optimisation work is often left implicit; verification evidence, including observed convergence orders on non-smooth models, should accompany it. Third, claims for surrogate assistance should be accompanied by equal-budget comparisons against simple baselines. The present study addresses these gaps within the limits of an assumed-parameter case.

Table 2. Representative prior work and the gap addressed in this study

Ref.	Focus / method	Key contribution	Gap relative to this study
[5]	Electrical energy of manufacturing processes	Fixed (idle) power makes specific energy fall as processing rate rises	Process-level and static; no dynamics, tariffs or optimisation
[6]	Review of efficiency at machine, line and factory levels	Systems view linking scheduling, idle time and equipment	Survey; little quantitative guidance on dynamic models or trade-offs
[2]	Energy-management standard	Baselines, indicators and continual improvement	Prescribes practice, not a method for choosing operating points
[10]	NSGA-II elitist Pareto search	Fast non-dominated sorting with crowding distance	Generic; needs a verified plant simulator and an evaluation-budget strategy
[9]	Surrogate-assisted evolutionary computation	Fewer expensive evaluations via approximate models	Value depends on evaluation cost; tested here against equal-budget random sampling
[19], [20]	Physics-informed neural networks	Equation residuals embedded in the training loss	Needs measured data; ODE structure here is explicit, so deferred to future work
[22]	SINDy equation discovery	Interpretable ODEs from time-series data	Needs rich plant measurements that are not yet available
[23]	Neural ODEs	Learned vector fields with adaptive solvers	Less transparent for energy audits than a physics-based model

[24]	Model predictive control	Constrained receding-horizon optimal control	Requires a validated model and online computation
This study	ODE model, verified solvers, surrogate, NSGA-II, sensitivity, robustness	Integrated, reproducible workflow for operating-point selection	Parameters assumed, not measured; plant validation still required

METHODOLOGY

A. Study Design and System Description

The study is a computational experiment on a representative electrically heated batch-processing line: a thermal processing zone (an oven, dryer or curing chamber) fed by a conveyor driven by a variable-speed motor. The line is electrically heated so that the energy balance is purely electrical, which ties energy and cost directly to a time-of-day tariff. It serves an order stream with a pre-existing backlog. Three operating variables are chosen by the plant manager: the temperature setpoint T_{set} , a base line speed u_{base} expressed as a fraction of full speed, and a load-shifting fraction ϕ that moves speed from the peak tariff window to off-peak windows. Fig. 1 shows the workflow, from model construction and solver verification through sampling, surrogate learning and search to verification and decision support.

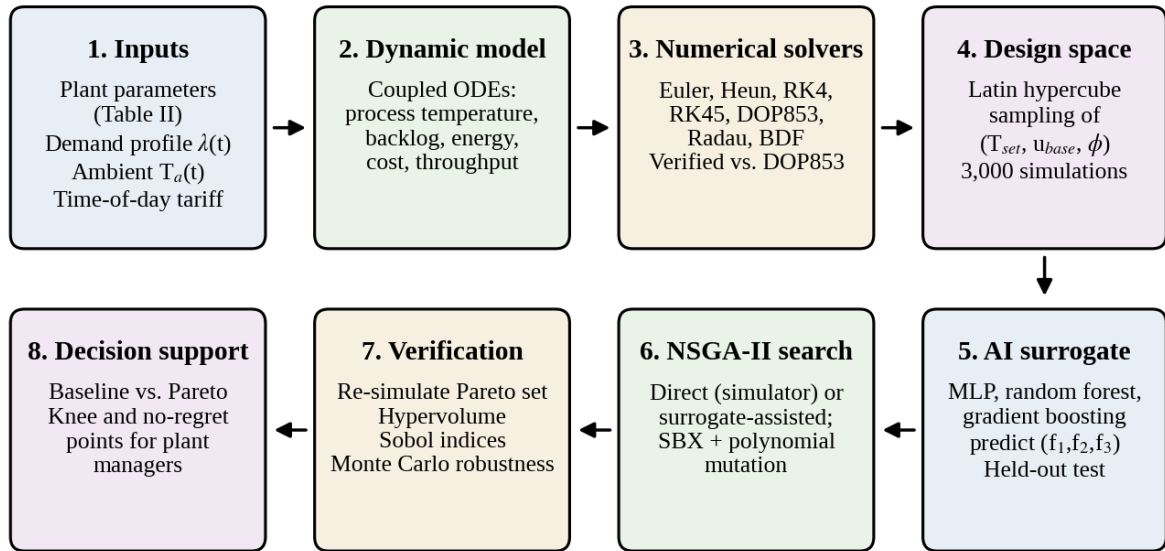


Fig. 1. Computational framework of the study: from model inputs and numerical verification to surrogate-assisted and direct multi-objective search, and finally to decision support.

B. Governing Equations

Let the state vector be $y = [T, Q, E, J, N, I]^T$, where T is the process temperature ($^{\circ}\text{C}$), Q the order backlog (units), E the cumulative electrical energy (kWh), J the cumulative electricity cost (NPR), N the cumulative number of units processed, and I the time integral of the backlog (unit-hours). The thermal balance is

$$\frac{dT}{dt} = \frac{\eta P_h - (k_l + \gamma \mu)(T - T_a(t))}{C} \quad (1)$$

where P_h is the heater electrical power, η the heater efficiency, k_l the loss coefficient to ambient, γ the product-heating coefficient, μ the throughput, $T_a(t)$ the ambient temperature and C the thermal capacity. The first term is heat delivered, the second is loss to ambient, and the third is the heat absorbed by the product, which grows with throughput. The backlog evolves as

$$\frac{dQ}{dt} = \lambda(t) - \mu(t) \quad (2)$$

with arrival rate $\lambda(t)$. Throughput depends on line speed, temperature and the availability of work:

$$\mu(t) = \mu_{max} u(t) s(T) \tanh\left(\frac{Q}{q_0}\right) \quad (3)$$

$$s(T) = \frac{1}{1 + \exp\left(-\frac{T - T_c}{w}\right)} \quad (4)$$

Here μ_{max} is the maximum throughput, $u(t)$ the speed fraction, and $s(T)$ a logistic quality factor that suppresses acceptable output when the zone is colder than the process requires (midpoint T_c , width w). The factor $\tanh(Q/q_0)$ is a smooth starvation term: when the backlog approaches zero the line cannot process more than arrives. The heater follows a saturated proportional controller, and total electrical power adds a motor term with an idle component and a cubic speed-dependent component, which is a stylised form of the power-speed relation of variable-speed drives:

$$P_h = \min\{P_{max}, \max\{0, K_p(T_{set} - T)\}\} \quad (5)$$

$$P = P_h + P_0 + P_r u^3 + P_{aux} \quad (6)$$

The remaining states are quadratures that accumulate the quantities needed for the objectives:

$$\frac{dE}{dt} = P, \quad \frac{dJ}{dt} = c(t)P, \quad \frac{dN}{dt} = \mu, \quad \frac{dI}{dt} = Q \quad (7)$$

where $c(t)$ is the time-of-day tariff in NPR/kWh. The model is deliberately compact: it has six states, a handful of interpretable parameters, and enough structure (saturation, starvation, threshold-type quality, piecewise-constant schedules) to make solver choice and optimisation non-trivial. Table II lists all parameters.

C. Operating Policy, Tariff and Scenario

Speed follows a schedule that depends on the tariff window. The base speed is scaled by a multiplier $m(t)$ and clipped to the physically allowed range:

$$u(t) = \text{clip}(u_{base}m(t), 0.2, 1), \quad m(t) = \begin{cases} 1 + \kappa\varphi & \text{(off-peak)} \\ 1 & \text{(normal)} \\ 1 - \varphi & \text{(peak)} \end{cases} \quad (8)$$

The tariff has three bands: off-peak 22:00–06:00, normal 06:00–18:00 and peak 18:00–22:00, priced at 6.0, 8.5 and 11.5 NPR/kWh respectively. These values are a stylised three-band structure of the kind used in time-of-day pricing; they are assumed for illustration and are not the official Nepal Electricity Authority schedule, which should replace them in any applied use. The arrival rate is $\lambda(t) = 30 + 20 \exp[-((t - 13)/4)^2]$ units/h, giving roughly 862 units of new demand per day on top of an initial backlog of 100 units, and ambient temperature follows $T_a(t) = 24 + 7 \sin(2\pi(t - 9)/24)$ °C. With the proportional gain in Table II, the closed-loop thermal time constant is $C/(K_p\eta) \approx 0.084$ h, about five minutes.

The conventional baseline is defined as $T_{set} = 215$ °C, $u_{base} = 0.85$ and $\varphi = 0$. It represents an operator who sets a high temperature for quality margin and runs close to full speed with no tariff awareness. This is an assumption made for comparison, not an observation of any particular plant.

Table 2. Parameters, schedule and decision-variable ranges of the case-study model (assumed values)

Parameter	Symbol	Value	Unit
Thermal capacity	C	2.0	kWh/°C
Loss coefficient to ambient	k_l	0.35	kW/°C
Product-heating coefficient	γ	0.004	kWh/(unit·°C)
Heater efficiency	η	0.95	–
Proportional gain	K_p	25	kW/°C
Heater capacity	P_{max}	150	kW
Motor idle / drive / auxiliary power	P_0, P_r, P_{aux}	6, 24, 3	kW
Maximum throughput	μ_{max}	60	units/h
Quality midpoint and width	T_c, w	175, 7	°C
Starvation scale	q_0	3	units

Initial backlog and temperature	Q_0, T_0	100, 140	units, °C
Load-shift gain (off-peak)	κ	0.6	–
Tariff: off-peak / normal / peak	$c(t)$	6.0 / 8.5 / 11.5	NPR/kWh
Setpoint range	T_{set}	170–240	°C
Base speed range	u_{base}	0.5–1.0	fraction
Load-shift fraction range	ϕ	0–0.6	fraction

D. Numerical Integration and Verification

Writing the system as $y' = f(t, y; \theta)$, the study compares explicit Euler, Heun (second-order), classical RK4, the adaptive Dormand–Prince pair RK45 [11], the eighth-order DOP853 method [7], and the implicit Radau and BDF methods [14], the last four through SciPy [13]. Fixed-step methods were implemented directly in NumPy. Because the speed schedule is piecewise constant, integration is carried out segment by segment between 06:00, 18:00 and 22:00, and every fixed step size divides each segment exactly, so that no step straddles a discontinuity. The RK4 update is

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4) \quad (9)$$

with $k_1 = f(t_n, y_n)$, $k_2 = f(t_n + \frac{h}{2}, y_n + \frac{hk_1}{2})$, $k_3 = f(t_n + \frac{h}{2}, y_n + \frac{hk_2}{2})$ and $k_4 = f(t_n + h, y_n + hk_3)$.

The reference solution uses DOP853 with relative and absolute tolerances of 10^{-12} . Two error measures are reported for the baseline operating point: the maximum absolute temperature error over 121 output times spaced 0.2 h apart, and the relative error in the 24-hour energy $E(24)$. Observed convergence order is estimated as the least-squares slope of log error against log step size over the four finest step sizes. Cost is measured by the number of right-hand-side evaluations and by the median wall time of five runs; because these are single-threaded interpreted-Python timings they are indicative only. For optimisation, a vectorised RK4 with $h = 1/30$ h (two minutes) advances all candidate designs at once, and its fidelity is checked against the reference on the baseline and on 30 independent Latin-hypercube designs.

E. Objective Functions

Three objectives, all to be minimised, are evaluated over the 24-hour horizon:

$$f_1 = \frac{E(24)}{N(24)}, f_2 = \frac{J(24)}{N(24)}, f_3 = \frac{I(24)}{1000} \quad (10)$$

Here f_1 is the specific energy consumption (kWh per unit), f_2 is the electricity cost per unit (NPR/unit) and f_3 is a delay index equal to the accumulated backlog in thousands of unit-hours, which grows when orders wait longer. Because most designs clear the backlog within the day, the number of units processed is nearly constant (about 960), so f_1 and f_2 mainly reflect daily energy and cost.

F. Sampling and Surrogate Models

Designs were sampled by Latin hypercube [31] over the ranges in Table II: 3,000 designs for training and an independent 1,000 for testing. Three surrogate families were trained with scikit-learn [32] to predict (f_1, f_2, f_3) from $(T_{set}, u_{base}, \phi)$: a multilayer perceptron (MLP, two hidden layers of 64 tanh units, Adam optimiser), a random forest of 300 trees, and gradient-boosted trees (400 trees of depth 3, learning rate 0.05, one model per objective). Inputs and outputs were standardised. Accuracy was measured by the coefficient of determination R^2 and the root-mean-square error (RMSE) on the held-out set, for training sizes of 300 and 3,000.

G. Multi-Objective Search

NSGA-II [10] was implemented in NumPy with simulated binary crossover (distribution index 15, crossover probability 0.9), polynomial mutation (index 20, probability 1/3) and a population of 100. Three strategies were compared. The *direct* strategy runs 60 generations on the simulator itself, using 6,100 simulator calls. The *surrogate-assisted* strategy simulates 300 Latin-hypercube designs, trains the MLP, runs 200 generations of NSGA-II on the surrogate, then re-simulates the final 100 solutions and retains the verified non-dominated ones, using 400 simulator calls. The *random baseline* simulates 400

Latin-hypercube designs and keeps the non-dominated subset. Each strategy was repeated with ten independent seeds.

Front quality is summarised by the hypervolume [29], computed exactly by slicing. Objectives are normalised between an ideal point (the componentwise minimum over all solutions found) and a reference point (1.02 times the componentwise maximum of the non-dominated set of the 3,000-design pool), so that hypervolume is a fraction of the normalised box. A compromise (knee) solution is the front member closest to the ideal point in normalised space, and a no-regret solution is the front member with the lowest energy among those that dominate the baseline in all three objectives.

H. Sensitivity and Robustness Analysis

Global sensitivity was assessed with first-order and total Sobol indices estimated with the Saltelli scheme [33], [34] on a scrambled Sobol sequence with 4,096 base samples, requiring 20,480 simulations, and with standardised regression coefficients (SRC) on the Latin-hypercube pool. Robustness was assessed with 2,000 Monte Carlo draws of the physical parameters from uniform distributions: $k_l \pm 15\%$, $\gamma \pm 20\%$, η between 0.92 and 0.98, $T_c \pm 5^\circ\text{C}$, $P_r \pm 15\%$, $\mu_{max} \pm 10\%$ and $P_0 \pm 15\%$. Each selected operating point was evaluated against the baseline using the same draws (a paired comparison), so that differences are not confounded by parameter sampling.

I. Implementation and Reproducibility

All computations used Python 3.12 with NumPy, SciPy and scikit-learn on a single CPU, with fixed random seeds. All parameters are listed in Table II and every equation is stated in this section, so the study can be reproduced without proprietary data. The authors have not tuned the NSGA-II or surrogate hyper-parameters, and no results were discarded.

RESULTS AND ANALYSIS

A. Baseline Dynamics

Fig. 2 shows the 24-hour response at the conventional baseline. Starting from 140°C , the heater runs saturated at 150 kW for about 2.2 h, and total electrical draw during warm-up reaches about 174 kW. Temperature then settles at 211.2°C , 3.8°C below the 215°C setpoint, which is the steady-state offset expected from proportional-only control; setpoint and achieved temperature therefore have to be distinguished in any energy audit. At that temperature the quality factor is about 0.994, so the line is limited by speed and demand rather than by temperature.

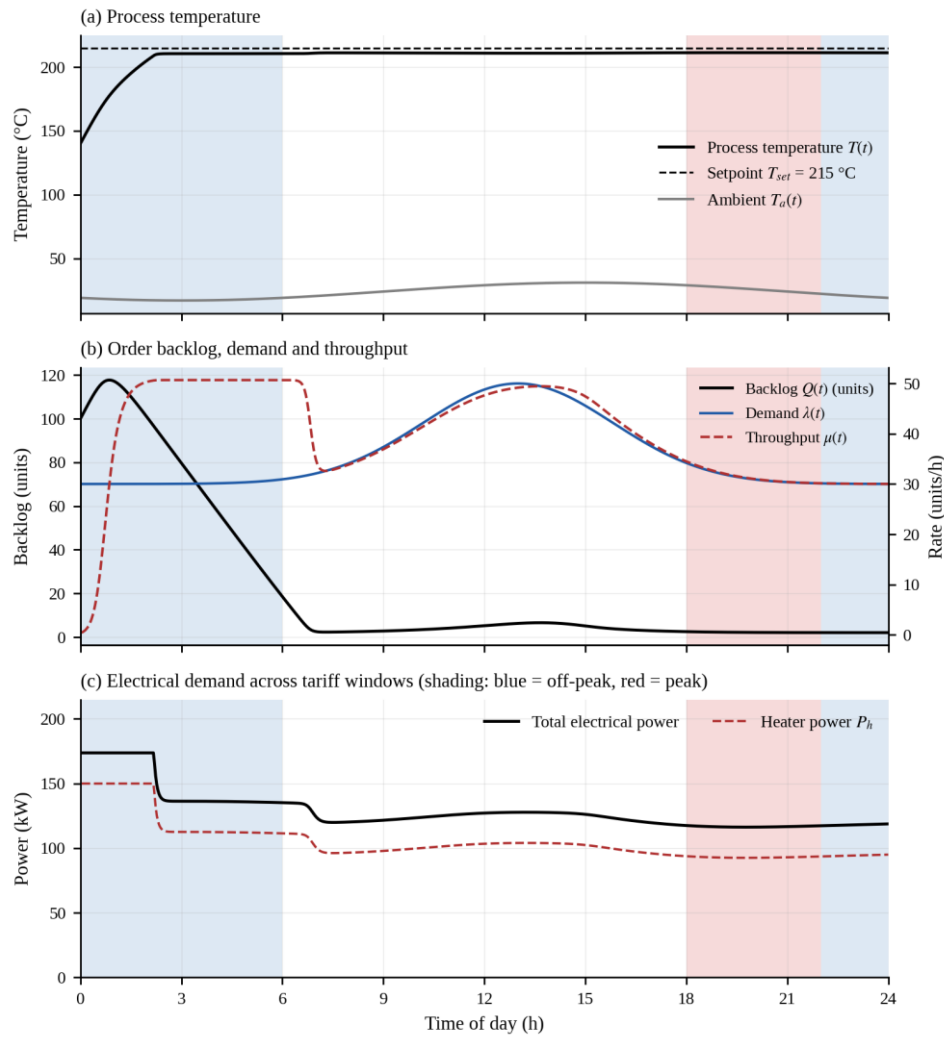


Fig. 2. Simulated 24-hour response at the conventional baseline ($T_{set} = 215\text{ }^{\circ}\text{C}$, $u_{base} = 0.85$, $\varphi = 0$): (a) process temperature, (b) backlog, demand and throughput, (c) electrical power. Shading marks off-peak (blue) and peak (red) tariff windows. Source: authors' simulation with parameters from Table II.

In the off-peak hours the line's capacity (about 50.7 units/h) exceeds demand (about 30 units/h), so the backlog drains from 100 units to about 3 units by roughly 07:00 (Fig. 2b). Thereafter throughput simply tracks demand, because the starvation factor prevents the extra capacity from producing additional output. This is the main source of waste in the scenario: the heater continues to hold $211\text{ }^{\circ}\text{C}$ and the motors continue to draw idle power while the line is demand-limited. Electrical power falls from roughly 136 kW late in the off-peak period to about 120 kW after 07:00 (Fig. 2c), because the product-heating load falls with throughput. Daily totals are 3098.6 kWh, 959.7 units and NPR 24,894, which gives $f_1 = 3.229\text{ kWh/unit}$, $f_2 = 25.94\text{ NPR/unit}$ and $f_3 = 0.525$.

B. Solver Verification

Table III and Fig. 3 answer RQ1. Each method is compared with the DOP853 reference at the baseline operating point. Observed convergence orders over the four finest step sizes are 1.04 for Euler, 2.16 for Heun and 1.64 for RK4. Euler and Heun therefore behave as theory predicts, but RK4 falls well short of its theoretical order of four. This is consistent with the loss of smoothness at heater-saturation transitions, although the present experiments did not isolate that cause. The practical lesson is that the nominal order of a method should not be used to extrapolate its accuracy on a non-smooth plant model.

Table 3. Solver verification against the DOP853 reference ($\text{rtol} = \text{atol} = 10^{-12}$) at the baseline operating point

Method	Setting	Max $ \Delta T $ (°C)	Rel. error in $E(24)$	RHS evaluations	Wall time (ms)
Euler	$h = 0.1$ h	9.80×10^{-1}	1.99×10^{-4}	240	2.8
Euler	$h = 0.00625$ h	5.79×10^{-2}	1.21×10^{-5}	3840	37.8
Heun (RK2)	$h = 0.05$ h	7.28×10^{-2}	6.32×10^{-7}	960	10.4
RK4	$h = 0.2$ h	6.30×10^{-1}	2.43×10^{-6}	480	5.0
RK4	$h = 0.05$ h	3.16×10^{-3}	9.72×10^{-8}	1920	19.9
RK45 (Dormand-Prince)	$\text{rtol} = 10^{-3}$	3.08×10^0	4.86×10^{-3}	818	17.7
RK45 (Dormand-Prince)	$\text{rtol} = 10^{-6}$	7.62×10^{-4}	2.03×10^{-8}	1070	20.7
DOP853	$\text{rtol} = 10^{-6}$	3.53×10^{-2}	6.90×10^{-11}	1379	26.2
Radau (implicit)	$\text{rtol} = 10^{-6}$	8.61×10^{-6}	5.93×10^{-10}	1589	82.2
BDF (implicit)	$\text{rtol} = 10^{-6}$	2.77×10^{-4}	2.76×10^{-7}	889	56.2

Several findings stand out. At $h = 0.2$ h (12 minutes) Euler and Heun give temperature errors of about 4.9 and 4.1 °C, because the step exceeds roughly twice the five-minute closed-loop time constant, which is the approximate stability limit of both methods, whereas RK4, with a wider stability region, stays within 0.63 °C. In the work–precision plot (Fig. 3b), RK4 is more economical than Euler by more than an order of magnitude at errors below about 0.1 °C, and RK45 at a tolerance of 10^{-6} reaches 7.6×10^{-4} °C with 1,070 evaluations, whereas RK4 needs about 3,840 evaluations ($h = 0.025$ h) to reach 1.2×10^{-3} °C. The adaptive solver, however, is sensitive to its tolerance: at 10^{-3} , which is the default relative tolerance of SciPy’s *solve_ivp*, the temperature error is 3.1 °C, and the error is not strictly monotone in tolerance (10^{-7} gives a larger error than 10^{-6}). This is typical when local error control is applied to a non-smooth right-hand side, since a local tolerance does not guarantee a global error bound.

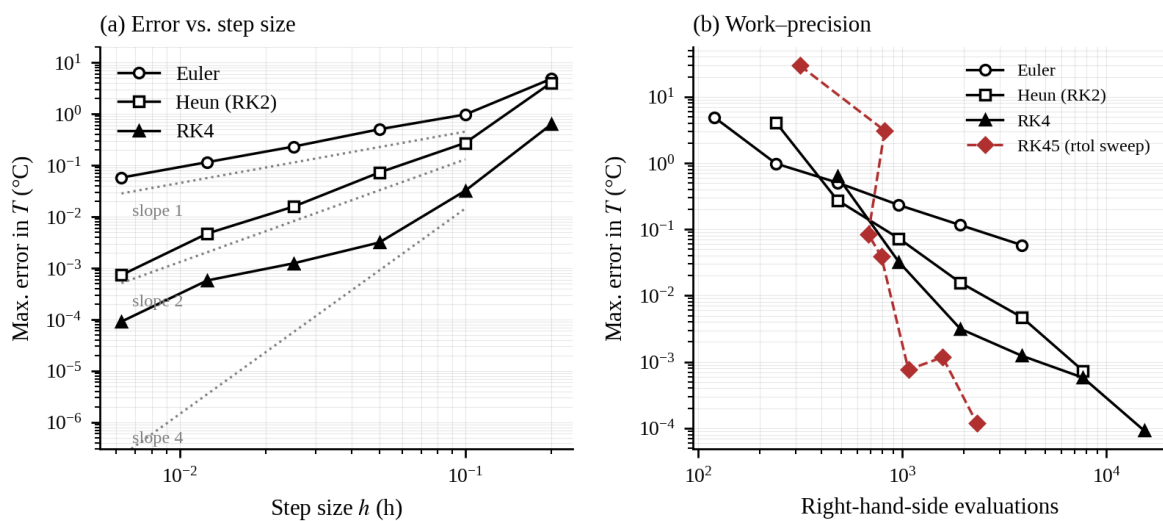


Fig. 3. Solver verification at the baseline operating point: (a) maximum temperature error versus step size for Euler, Heun and RK4 with dotted lines indicating ideal slopes; (b) work–precision diagram showing error versus right-hand-side evaluations, including a tolerance sweep of RK45. Source: authors’ simulation.

The implicit methods deliver high accuracy (Radau 8.6×10^{-6} °C) but at 56–82 ms they are three to four times slower than RK45 at 18–21 ms, which indicates that the model is not stiff enough to benefit

from them: the fastest time scale is five minutes against thermal and production dynamics of hours. The DOP853 temperature error at 10^{-6} ($0.035\text{ }^{\circ}\text{C}$) is larger than its energy error (6.9×10^{-11}); a plausible reason is interpolation at the output times across the saturation kink, but this was not tested. For the optimisation experiments, the vectorised RK4 with a two-minute step deviated from the reference baseline objectives by 6.0×10^{-8} , 4.4×10^{-8} and 1.3×10^{-7} (relative) for f_1 , f_2 and f_3 , and by at most 5.5×10^{-6} , 5.7×10^{-6} and 3.4×10^{-5} across 30 random designs. These numerical errors are orders of magnitude smaller than the surrogate and parameter uncertainties discussed next, so the optimisation is not distorted by integration error.

C. Surrogate Accuracy

Table IV and Fig. 4(a)–(c) answer the first half of RQ2. The MLP is the most accurate surrogate at both training sizes. With 3,000 training designs it achieves R^2 of 0.9989, 0.9991 and 0.9994 for the three objectives, with RMSE of 0.027 kWh/unit, 0.199 NPR/unit and 0.058 on the delay index. With only 300 designs the MLP still reaches $R^2 \geq 0.997$, whereas the random forest and gradient-boosting models need about ten times as much data to approach the same level ($R^2 \approx 0.998$ at 3,000 designs). This ordering is plausible because the objectives are smooth functions of three inputs, which suits a neural network, whereas tree ensembles build piecewise-constant approximations.

Table 4. Surrogate accuracy on 1,000 held-out designs

Model	Training designs	$R^2(f_1)$	$R^2(f_2)$	$R^2(f_3)$	RMSE (f_1)	RMSE (f_2)	RMSE (f_3)
MLP	300	0.9975	0.9979	0.9985	0.042	0.303	0.097
MLP	3,000	0.9989	0.9991	0.9994	0.027	0.199	0.058
Random forest	300	0.9832	0.9841	0.9901	0.108	0.843	0.246
Random forest	3,000	0.9979	0.9982	0.9986	0.038	0.284	0.092
Gradient boosting	300	0.9906	0.9912	0.9944	0.081	0.628	0.185
Gradient boosting	3,000	0.9979	0.9979	0.9982	0.039	0.303	0.104

Global accuracy can nevertheless overstate usefulness for optimisation. Across the whole design space the objectives span 2.83–8.94 kWh/unit, so an RMSE of 0.027 kWh/unit is under 0.5% of the range. Near the Pareto front, however, f_1 spans only about 0.96 kWh/unit, and the same RMSE is about 2.8% of that span; the corresponding figures for f_2 and f_3 are about 2.5% and 5.5%. Errors of this size are comparable to the spacing between competing near-front designs and can therefore reorder them. As regards cost, simulating 3,000 designs with the vectorised RK4 took 0.35 s (about 0.12 ms per design), whereas the MLP predicted 10,000 designs in 0.0125 s (about 1.2 μs per design), roughly 100 times faster. Against a scalar adaptive solve of 18–21 ms (Table III) the speed-up is about four orders of magnitude, but because the vectorised simulator is itself cheap, the absolute saving in this model is small.

D. Global Sensitivity

Fig. 4(d) answers RQ3 in part. The setpoint temperature dominates every objective: its first-order Sobol indices are 0.894 for f_1 , 0.892 for f_2 and 0.761 for f_3 , with total indices of 0.933, 0.931 and 0.776. Base speed contributes 0.067, 0.069 and 0.220 (total 0.106, 0.109, 0.236), so it matters mostly for delay, and the small gap between total and first-order indices points to modest interaction with the setpoint. The load-shift fraction is almost negligible over the whole design space, with first-order indices of at most 0.002 and total indices of at most 0.005.

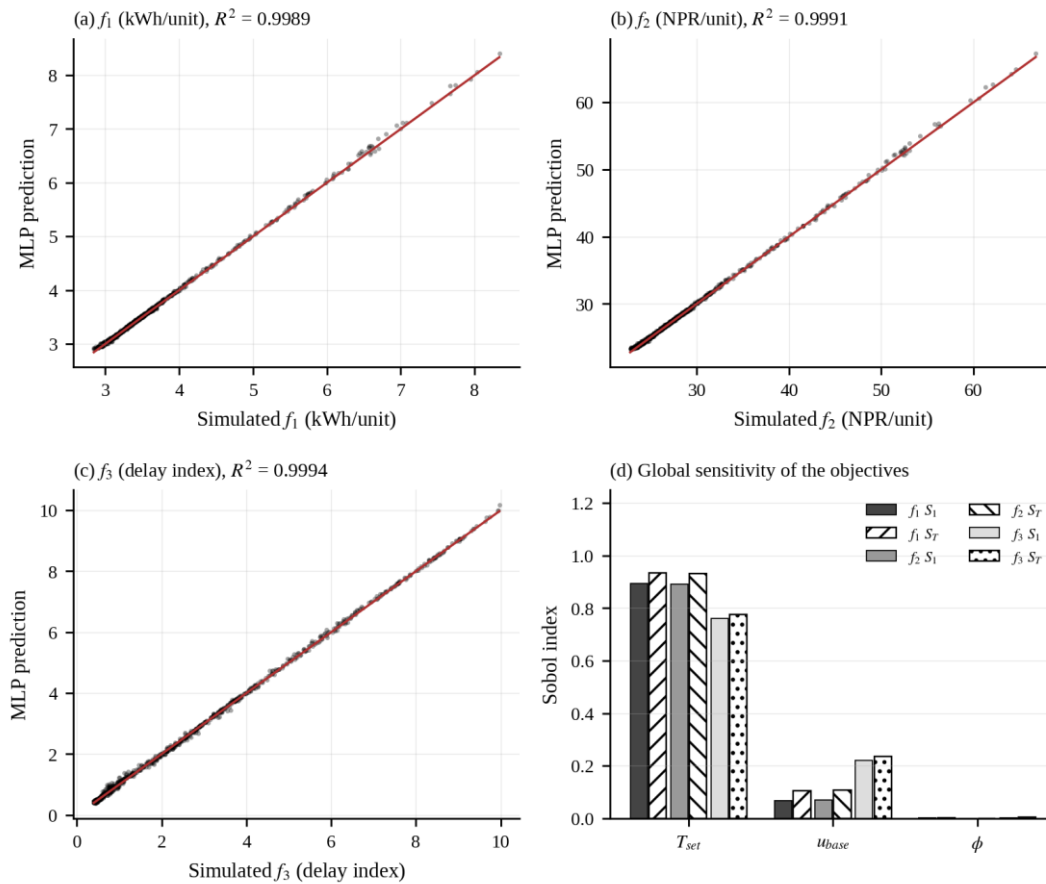


Fig. 4. Surrogate and sensitivity results: (a)–(c) parity plots of the MLP trained on 3,000 designs against simulator values on 1,000 held-out designs for f_1 , f_2 and f_3 ; (d) first-order (S_1) and total (S_T) Sobol indices of the decision variables. Source: authors' simulation.

Linear regression performs poorly as a screening tool here. The SRC fits explain only 23% of the variance of f_1 and f_2 and 66% of f_3 , which indicates strongly non-monotone behaviour. This is consistent with a U-shaped response of energy to setpoint, in which low setpoints starve the line through the quality factor and high setpoints waste heat, but the present analysis does not decompose the response further.

E. Pareto Optimisation

Fig. 5 and Table V present the multi-objective results. The direct NSGA-II run with seed 0 (6,100 simulator calls) returns 100 non-dominated solutions, spanning 2.824–3.789 kWh/unit in f_1 , 22.64–30.46 NPR/unit in f_2 and 0.398–1.447 in f_3 . In the energy–cost projection (Fig. 5a) the front is almost a straight line, because cost is essentially the energy multiplied by a tariff of roughly 8 NPR/kWh. The baseline lies close to this line but displaced toward higher energy, and in the energy–delay projection (Fig. 5b) it sits inside the dominated region: 20 of the 100 front solutions dominate it in all three objectives, and 59 do not worsen delay.

The decision-space view in Fig. 5(c) shows two regimes. For setpoints of roughly 191–205 °C, speeds are 0.78–0.96 and load shifting is active (ϕ up to 0.37), and these solutions trade delay for energy. For setpoints above roughly 206 °C, the speed saturates at its upper bound and the load-shift fraction declines toward zero as the setpoint rises (it is close to zero above about 215 °C), so these solutions reduce delay at the expense of energy. Table V reports representative members of the front.

Table 5. Selected operating points from the direct NSGA-II Pareto set (seed 0), verified with the simulator

Operating point	T_{set} (°C)	u_{base}	ϕ	f_1	f_2	f_3	Δf_1 vs. baseline (%)	Δf_2 vs. baseline (%)
Conventional baseline	215.0	0.85	0.00	3.229	25.94	0.525	—	—

Minimum energy	192.2	0.79	0.00	2.824	22.83	1.445	-12.5	-12.0
Minimum cost	192.1	0.78	0.25	2.845	22.64	0.974	-11.9	-12.7
Minimum delay	240.0	1.00	0.00	3.789	30.46	0.398	+17.4	+17.4
Compromise (knee)	193.7	0.85	0.22	2.949	23.48	0.545	-8.6	-9.5
No-regret	192.2	0.89	0.29	2.968	23.57	0.523	-8.1	-9.1

The compromise solution ($T_{set} = 193.7$ °C, $u_{base} = 0.85$, $\phi = 0.22$) lowers energy per unit by 8.6% and cost per unit by 9.5%, while the delay index rises by 3.8%. In daily terms this is 2830.8 kWh against 3098.6 kWh and NPR 22,532 against NPR 24,894, with essentially the same output (959.8 versus 959.7 units). The cost saving exceeds the energy saving because mean power in the peak window falls from 116.7 to 98.4 kW (a reduction of 15.7%). The no-regret solution ($T_{set} = 192.2$ °C, $u_{base} = 0.89$, $\phi = 0.29$) lowers energy by 8.1% and cost by 9.1% with a delay index 0.4% lower than the baseline.

The extreme solutions illustrate the trade-offs. The minimum-energy design cuts energy by 12.5% but raises the delay index to 1.445, about 2.75 times the baseline, because it runs a low setpoint at 0.785 speed. The minimum-delay design cuts the delay index by 24% but at the setpoint upper bound of 240 °C and full speed, costing 17.4% more energy. Comparing the minimum-energy and minimum-cost designs isolates the effect of load shifting: allowing $\phi = 0.25$ lowers cost by 0.85% at a 0.74% energy penalty, while reducing the delay index by 33% (from 1.445 to 0.974). In this model, load shifting therefore buys timeliness at almost no energy cost, rather than large cost savings.

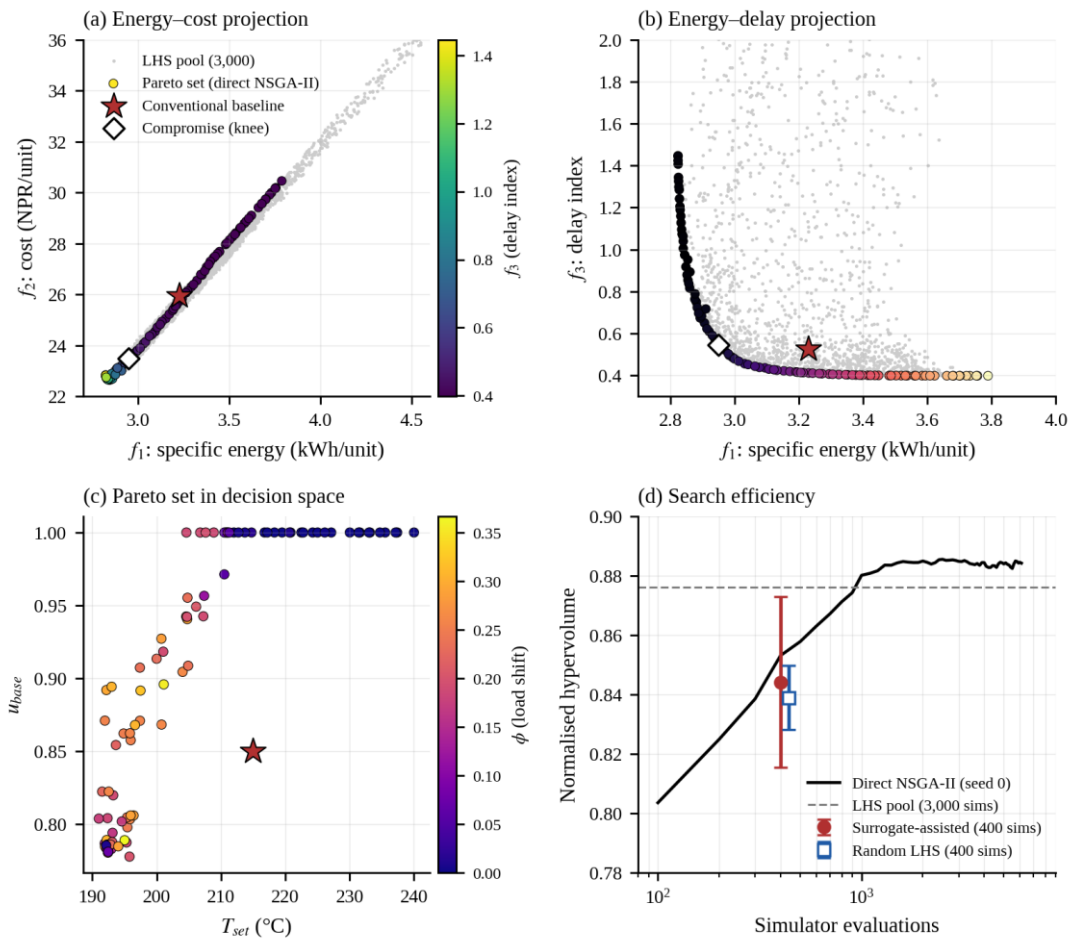


Fig. 5. Multi-objective results: (a) energy–cost and (b) energy–delay projections of the direct NSGA-II front (seed 0) with the Latin-hypercube pool, baseline and compromise solution; (c) the Pareto set in decision space; (d) normalised hypervolume versus simulator evaluations for direct search, with mean $\pm$ one standard deviation (ten seeds) for the 400-evaluation strategies. Source: authors’ simulation.

F. Search Efficiency

Fig. 5(d) and the associated hypervolume statistics address the second half of RQ2. Direct NSGA-II reaches a normalised hypervolume of 0.8836 ± 0.0014 (mean $\pm$ standard deviation over ten seeds) with 6,100 simulator calls, and it passes the value of the 3,000-design Latin-hypercube pool (0.876) after roughly 1,000 evaluations. At a budget of 400 simulations, the surrogate-assisted strategy gives 0.844 ± 0.029 and plain random Latin-hypercube sampling gives 0.839 ± 0.011 . The difference between these two is small relative to the run-to-run spread, so the present ten-seed experiment does not support a claim that surrogate assistance outperforms random sampling at equal budget. Surrogate assistance did produce denser verified fronts (47–81 non-dominated solutions per run against 26–34 for random sampling) but with considerable variability in quality, from a hypervolume of 0.775 in the worst seed to 0.873 in the best, depending on the training sample.

A hypothesis for this outcome, which was not tested, is that surrogate errors of 2.5–5.5% of the front's span (Section IV-C) are large enough to reorder near-front designs, so the search converges to a front that is good on the surrogate but only partly good on the simulator. A second contributing factor is that the design space has only three dimensions, so even random sampling covers it well; the Latin-hypercube pool alone attains a hypervolume of 0.876, within 1% of the best direct result. The benefit of sophisticated search and of surrogates would be expected to grow with the number of decision variables and with the cost of each evaluation.

G. Robustness to Parameter Uncertainty

Under the paired Monte Carlo test with 2,000 parameter draws, the baseline has mean objectives of 3.234 ± 0.179 kWh/unit, 25.99 ± 1.46 NPR/unit and a delay index of 0.543 ± 0.078 . The compromise solution achieves 2.962 ± 0.166 kWh/unit and 23.58 ± 1.35 NPR/unit, and it beats the baseline on both energy and cost in 100% of the draws. Its delay outcome is much less secure: the mean delay index is 0.660 with a standard deviation of 0.295, and it beats the baseline on delay in only 38% of draws. The no-regret point shows the same pattern: energy and cost are better than the baseline in all draws, but delay is better in only 49% of them, which is also the share in which the point dominates the baseline outright. The minimum-energy and minimum-cost designs beat the baseline on energy and cost in 97–98.5% of draws but are worse on delay in every draw, and the minimum-delay design is better on delay in every draw and worse on energy and cost in every draw.

The energy and cost improvements are thus robust to the assumed parameter ranges, whereas the delay performance of lean operating points is sensitive to them. A one-factor check around the no-regret point supports this reading and shows an asymmetry. Raising the setpoint by 2 °C or 5 °C increases energy per unit by 1.0% or 2.5% and lowers the delay index by 8% or 14%; lowering it by 2 °C or 5 °C reduces energy by 1.0% or 2.2% but raises the delay index by 14% or 86%. A small safety margin on the high side is therefore cheap compared with the delay penalty of undershooting.

DISCUSSION

The results answer the three research questions with different degrees of confidence. For RQ1, fixed-step RK4 at a two-minute step is both accurate and cheap for this model, and the adaptive and implicit alternatives offer no advantage for the optimisation workload; the more important finding is methodological, namely that observed convergence orders can differ markedly from nominal orders on non-smooth plant models, and that a default adaptive tolerance of 10^{-3} can produce temperature errors of several degrees. Reporting the solver, its settings and verification evidence should be part of any ODE-based energy study. For RQ2, surrogates are accurate in the global sense, but at equal budget they did not outperform random sampling here. For RQ3, the setpoint is the dominant lever, and the energy and cost gains from changing it are robust to parameter uncertainty.

Why does the setpoint dominate? In the model, heat loss grows linearly with the temperature excess over ambient, whereas the quality factor saturates once the zone is comfortably above its midpoint. Beyond roughly 195–200 °C additional temperature buys almost no additional throughput and simply wastes heat, which is why the lean designs on the front sit near 192–194 °C and the baseline at 215 °C does not. This reading is specific to the assumed quality threshold ($T_c = 175$ °C), and in a real plant the corresponding threshold must come from product-quality specifications, not from an optimiser.

The practical message for Nepalese plants is the form of the analysis: temperature margins that are added for safety have a measurable energy price that can be weighed against the delay and quality risk of operating closer to the threshold.

The result is partly consistent with the earlier literature on idle power [5], but it adds a qualification. The established rule that faster processing lowers specific energy holds when there is work to process. In this scenario the minimum-energy design runs at 0.785 of full speed, not at full speed, because after the backlog has drained the line is demand-limited and running faster only starves the line while idle power is still drawn. The rule therefore needs to be applied together with an explicit model of demand, which is the kind of interaction that a dynamic, system-level model [6] can capture and a static process characterisation cannot.

Tariff-aware load shifting turned out to be a second-order lever in this model. The Sobol indices for the shift fraction are below 0.5%, energy and cost are almost collinear, and load shifting mainly reduced the delay index rather than the bill. Two features of the scenario explain this: the assumed ratio between peak and off-peak prices is only about 1.9, and the line is demand-limited for most of the day, so there is little deferrable load to move. A stronger price differential or a plant with buffer capacity and genuinely deferrable work might change the picture, but that is a hypothesis that this study did not test. For utilities, the implication is that price signals alone may not move loads that have little flexibility.

On the role of AI, the surrogate results are a cautionary tale about metrics. A global R^2 above 0.998 suggests an almost perfect model, yet the same error expressed relative to the span of the Pareto front is several per cent. Consistent with the general view that surrogate assistance pays off when evaluations are expensive and the surrogate is accurate where it matters [9], the benefit was not visible here, where evaluations cost fractions of a millisecond. This does not argue against surrogates in general. It argues for equal-budget baselines, for error measures that are local to the region of interest, and for using surrogates where the underlying simulator is costly, such as discrete-event or computational-fluid-dynamics models. Physics-informed and equation-discovery approaches [19], [22] would become relevant once measured data are available to calibrate the lumped model.

Finally, the work has a practical reading for plants operating under ISO 50001-type programmes [2]. The workflow provides an energy performance indicator (specific energy consumption), a baseline operating point, a set of alternatives with quantified trade-offs, and an uncertainty statement, all computed with open-source tools and modest computing. Before any of the numerical conclusions are used for decisions, the model must be calibrated on measured temperature, power and production data from the plant concerned.

LIMITATIONS

1. **Assumed parameters.** No plant measurements were used. All parameters, the demand profile, the ambient profile and the tariff are assumptions, so every numerical result is illustrative and no claim is made that Nepalese plants would achieve the reported percentage savings. Calibration and validation against measured data are the most important missing steps.

2. **Model structure.** The model has a single lumped thermal zone, a proportional-only controller, deterministic demand and ambient conditions, and electrical heating only. It omits product quality defects, start-up and shutdown scheduling, machine failures, grid interruptions, and thermal processes fuelled by biomass, coal or oil. An emissions objective, which is relevant to national climate goals [4], is not included.

3. **Stylised tariff.** The three-band tariff is illustrative, and demand charges, seasonal hydropower availability and real utility rules are not represented.

4. **Scope of the surrogate study.** Only one simulator, three decision variables and ten random seeds were used. Hyper-parameters of NSGA-II and the surrogates were not tuned, hypervolume depends on the chosen reference point, and no formal significance tests were run, so conclusions about surrogate assistance should not be extended to higher-dimensional or more costly problems.

5. **Scope of the solver study.** Convergence was examined at a single operating point, timings come from interpreted Python and are indicative, and the explanations offered for sub-theoretical orders and for DOP853 temperature errors were not isolated experimentally.

6. **Literature coverage.** The review is selective rather than systematic. References from 2024–2026 are limited to sources whose bibliographic details could be confirmed without database access, and a database-backed update of the recent literature is advisable before publication.

FUTURE SCOPE

1. **Calibration and field validation.** Instrument one or more Nepalese plants with smart meters, thermocouples and production logs, estimate the parameters of the lumped model, and compare predictions with measured behaviour. Sparse regression approaches such as SINDy [22] could suggest missing terms.

2. **Hybrid physics–machine-learning models.** Use physics-informed networks or neural ODEs [19], [23] to learn residual dynamics that the lumped model cannot capture, while keeping the interpretable physical core.

3. **Better control and scheduling.** Replace proportional control with PI or model predictive control [24] and optimise schedules in real time against forecast demand and tariffs.

4. **Richer objectives and uncertainty.** Add emissions, product quality, demand charges and seasonal supply conditions, adopt many-objective algorithms [26], [27] and use robust or stochastic formulations instead of post-hoc Monte Carlo checks.

5. **Costly simulators and focused surrogates.** Test surrogate assistance on discrete-event or computational-fluid-dynamics models, use adaptive infill criteria [30], and train surrogates to be accurate near the Pareto front rather than globally.

6. **Sector studies and decision support.** Extend the approach to brick, food-processing, textile and construction-material plants, and package the verified workflow as a simple tool that small and medium enterprises can use under energy-management programmes.

CONCLUSION

This paper developed a six-state ODE model of an electrically heated batch-processing line and used it to examine three questions that arise when AI and numerical differential-equation methods are combined for energy optimisation. On the numerical side, fixed-step RK4 with a two-minute step reproduced reference objectives to within 3.4×10^{-5} relative error and was the most practical choice, while the observed convergence order of 1.64 for RK4 and temperature errors above 3 °C for a default adaptive tolerance show that solver behaviour on non-smooth plant models needs to be verified. On the AI side, an MLP surrogate reached $R^2 \geq 0.998$ from 3,000 simulations, but surrogate-assisted search with 400 simulations did not beat random sampling at equal budget (hypervolume 0.844 versus 0.839), and direct NSGA-II was the best performer.

On the operating side, the setpoint temperature explained 76–89% of output variance, and a compromise solution cut energy per unit by 8.7% and cost per unit by 9.5% against a conventional baseline, with a 3.8% increase in the delay index. A no-regret solution cut energy by 8.1% and cost by 9.1% without a delay penalty. The energy and cost gains held in all 2,000 parameter-uncertainty draws, whereas delay outcomes of lean operating points were uncertain, and a small margin above the lean setpoint is a cheap form of insurance. Tariff-aware load shifting mainly affected delay rather than cost under the assumed tariff.

All of these values come from a model with assumed parameters and should be treated as a demonstration of method, not as a forecast for any specific factory. The transferable contributions are the verified numerical workflow, the equal-budget comparison methodology and the structure of the trade-offs. The next step for Nepalese manufacturing research is to calibrate the model with measured plant data, after which the same workflow can support credible, plant-specific energy-management decisions.

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