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Understanding Students' Mathematical Thinking and Attitudes: The Role of Error Analysis and Constructive Pedagogical Support in Secondary Mathematics

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ABSTRACT

This thesis examines the understanding of students' mathematical thinking during mathematics lessons, the observation of their ways of reasoning while solving problems and completing mathematical exercises, and the organization of pedagogical approaches that correspond to their thinking processes. Students' attitudes toward mathematics, fear of making mistakes, difficulties in finding solutions independently, and the attitudes of adults around them are analyzed as important factors influencing their mathematical activity. The thesis also emphasizes the importance of teachers assessing students' responses not only in terms of whether the final answer is correct or incorrect, but also by identifying how students reasoned, which method they used to solve a problem, and at which stage they encountered difficulties. It is emphasized that mathematical errors should not be treated as a means of punishment, but rather as an opportunity to identify students' thinking processes and improve subsequent educational activities.

1. Introduction

In mathematics education, it is important not only for students to obtain correct answers to exercises and problems, but also to understand how they think, how they interpret a task, and what approaches they use to reach a solution. Students may demonstrate different ways of reasoning when learning mathematical concepts and solving problems. Therefore, mathematics lessons should not be limited to evaluating the final answer; it is also necessary to observe and understand the thinking process through which students arrive at their solutions.

Students' attitudes toward mathematics can significantly influence their participation in the learning process. Some students develop a fear of making mistakes, anxiety about receiving low grades, or a lack of confidence in their ability to solve mathematical tasks. Such experiences can reduce their willingness to think independently and their interest in solving problems. In particular, repetitive mechanical exercises, solving tasks according to fixed patterns, and negative reactions to students' mistakes may restrict the development of mathematical thinking. Liljedahl (2020) argues that these barriers can be addressed by building "thinking classrooms", in which the usual norms of instruction are changed so that students are no longer afraid to make mistakes, no longer simply copy procedures, and instead work through problems on their own.

Research also shows that the teacher's actions during a lesson determine whether students' reasoning becomes visible. Ellis et al. (2019) demonstrate that effective instructional decisions depend on specific "teacher moves" — actions that elicit students' mathematical thinking, make it visible to the class, and build on it directly. Teachers and parents also shape students' attitudes toward mathematics: describing a child as having weak mathematical abilities may contribute to a negative attitude toward the subject, whereas encouragement and attempts to understand students' mistakes can support their interest and engagement.

Despite this, in everyday school practice students' responses are still frequently judged only as "correct" or "incorrect", and errors are treated as grounds for a lower mark rather than as evidence of how the student reasoned. The relationship between assessment practices, students' fear of errors, the organization of tasks, and the influence of adults has not been sufficiently considered as a single system

in the context of secondary mathematics education in Uzbekistan (Yuldasheva, 2023a). This defines the relevance of the present study.

The subject of the study is the process of understanding and observing students' mathematical thinking during mathematics lessons, including their reasoning methods, approaches to solving problems and exercises, difficulties in finding solutions independently, and responses to mathematical errors.

The aim of the study is to provide a theoretical justification for pedagogical approaches that take into account the characteristics of students' mathematical thinking and treat errors as a source of information about that thinking. To achieve this aim, the following objectives were set:

– to examine students' attitudes toward mathematics and the factors that influence their mathematical activity;

– to identify the role of fear of making mistakes and lack of confidence in the learning process;

– to analyze students' errors as indicators of their thinking processes rather than merely incorrect results;

– to determine the importance of observing how students reason, which methods they use, and at what stages they encounter difficulties;

– to describe pedagogical approaches that encourage students to explain their reasoning, explore different solution methods, and think independently, with attention to teacher support and cooperation with parents.

2. Materials and Methods

The study is theoretical and analytical in nature. Its material consists of three groups of sources: (1) international research on mathematics teaching, in particular the concept of "thinking classrooms" (Liljedahl, 2020) and research on teacher moves that support student reasoning (Ellis et al., 2019); (2) methodological materials of the Ministry of Preschool and School Education of the Republic of Uzbekistan (2023) on developing students' positive attitudes toward mathematics, supporting independent thinking, and strengthening cooperation between teachers and parents; and (3) studies by Uzbek researchers on students' attitudes toward mathematics and the role of errors in learning (Yuldasheva, 2023a, 2023b).

The following methods were used: theoretical analysis of the scientific and methodological literature; comparison of the positions presented in international and national sources; systematization of the factors that influence students' mathematical activity; and generalization of the pedagogical conditions under which errors can be used constructively. The sources were analyzed along four dimensions: (a) the focus of assessment — the final answer or the reasoning process; (b) students' attitudes toward mathematics and their fear of errors; (c) the organization of mathematical tasks; and (d) the role of adults — teachers and parents — in shaping students' mathematical activity. The results of the analysis are presented according to these dimensions.

3. Results

3.1. Reasoning process versus final answer

The analysis showed that understanding students' mathematical thinking requires attention to the reasoning that takes place while they solve problems and exercises. In a mathematics lesson, the same task may be approached by students in different ways. Therefore, evaluating only the final answer does not always provide sufficient information about a student's actual understanding. A teacher needs to observe how the student interprets the task, selects a solution strategy, explains the reasoning, and responds when difficulties arise. Such observation makes it possible to identify the specific aspects of mathematical thinking that require further development.

3.2. Attitudes toward mathematics and fear of errors

Students' attitudes toward mathematics are closely connected with their participation in mathematical activities. A student who believes that mathematics is too difficult or that they are unable to learn it may become reluctant to take part in problem solving. Statements such as "I cannot learn mathematics" or "I am not good at mathematics" reflect a lack of confidence and a negative attitude toward the subject. Fear of making mistakes and concern about receiving low grades further reduce students' willingness to attempt unfamiliar tasks. Consequently, a learning environment in which students can make mistakes, reconsider their ideas, and try alternative solutions is an important condition for supporting mathematical thinking.

3.3. Organization of tasks and the diagnostic function of errors

The way mathematical tasks are organized influences students' opportunities to develop reasoning skills. If learners repeatedly complete similar exercises mechanically or follow a fixed solution pattern without understanding the underlying reasoning, their opportunities to analyze and independently construct solutions become limited. In contrast, tasks that require explanation, comparison of different approaches, and reflection encourage students to become more actively involved in mathematical thinking.

Errors play a particular role in this process. A wrong answer can help a teacher understand how a student approached a problem, at which stage the problem was misunderstood, and where the reasoning changed direction. Instead of immediately treating an error as a failure, the teacher can ask the student to explain the steps taken and identify the point at which the difficulty occurred. This allows the teacher to provide more appropriate support and gives the student an opportunity to reconsider the solution. Thus, mathematical errors become a source of information for both teaching and learning.

3.4. The role of teachers and parents

The teacher's communication style was found to be an important factor. Negative comments about a student's mathematical abilities may contribute to a loss of confidence and strengthen an already negative attitude toward mathematics. Conversely, encouraging students to continue searching for solutions, recognizing their efforts, and discussing errors constructively create more favorable conditions for learning.

Parents also influence children's attitudes toward mathematics. When adults express negative opinions about their own mathematical experiences, children may develop similar perceptions of the subject. Excessive assistance with homework can create a further difficulty if it prevents children from attempting to solve problems independently. At the same time, parental encouragement and appropriate support can strengthen students' confidence. The factors identified and the corresponding pedagogical responses are summarized in Table 1.

Table 1. Factors influencing students' mathematical activity and corresponding pedagogical responses

Factor	Possible negative effect	Pedagogical response
Focus of assessment	Only the final answer is evaluated; the student's actual understanding remains hidden	Observe how the student interprets the task, chooses a strategy, and explains the reasoning
Attitude and fear of errors	Low confidence, reluctance to attempt unfamiliar tasks	Create an environment in which mistakes and revisions of ideas are accepted
Organization of tasks	Mechanical repetition and fixed patterns limit independent reasoning	Use tasks that require explanation, comparison of approaches, and reflection
Treatment of errors	Errors are perceived as failure or grounds for punishment	Ask the student to explain the steps and locate the point of difficulty
Teacher's communication	Negative remarks about ability reinforce negative attitudes	Encourage continued search, recognize effort, discuss errors constructively
Parents' attitudes and help	Transfer of negative attitudes; excessive help replaces independent effort	Teacher–parent cooperation; support rather than replace the child's own effort

4. Discussion

The results indicate that attention to students' mathematical thinking requires a shift from focusing exclusively on correct answers toward examining the processes through which students understand, reason, solve, make mistakes, and revise their ideas. This conclusion is consistent with Liljedahl's (2020) position that changing the usual norms of instruction — so that students are no longer afraid of mistakes and do not merely copy procedures — creates conditions for genuine independent thinking. It also agrees with Ellis et al. (2019): through teacher moves that elicit and make visible students' reasoning, an error stops being just a wrong answer and becomes something the class can reason through together.

The findings further suggest that the factors identified do not act in isolation. Fear of errors is reinforced when assessment focuses only on the result, when tasks are mechanical, and when adults comment negatively on a child's abilities; conversely, constructive treatment of errors is effective only when it is supported by appropriate tasks and consistent messages from both teachers and parents. For this reason, cooperation between teachers and parents is important for identifying negative attitudes at an early stage and for creating consistent support for independent learning, as also emphasized in the national methodological materials (Ministry of Preschool and School Education of the Republic of Uzbekistan, 2023).

From a practical perspective, the results imply that teachers should (1) ask students to explain their solutions rather than only report answers; (2) treat an incorrect answer as a starting point for discussion; (3) include tasks that admit several solution methods; and (4) communicate with parents about how to support, rather than replace, children's independent efforts.

The study has limitations. It is based on theoretical analysis of the literature and methodological documents and does not include empirical data. The relationships described between assessment practices, attitudes, and mathematical activity therefore require empirical verification. Future research could include classroom observations, questionnaires on students' attitudes toward mathematics, and pedagogical experiments evaluating the effect of error-analysis techniques on students' reasoning and motivation in secondary schools of Uzbekistan.

5. Conclusion

Observing students' mathematical thinking and responding appropriately to their reasoning can make mathematics learning more meaningful. The analysis showed that the focus of assessment, students' attitudes and fear of errors, the organization of tasks, and the positions of teachers and parents are interrelated factors that shape students' mathematical activity. Mathematical errors should be treated not as a means of punishment but as an opportunity to identify students' thinking processes and to improve subsequent instruction. Teacher support, constructive treatment of errors, opportunities for independent problem solving, and cooperation between teachers and parents can help develop students' positive attitudes toward mathematics, strengthen their confidence, and foster independent mathematical thinking.

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